OPTION PRICING: INPUTS, MISSPECIFICATION AND ALTERNATIVE MODELS

by

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Introduction

An overview of option pricing appeared in a recent issue of this journal. This outlined the basis of the option pricing model developed by Black and Scholes and examined a number of practical methods of estimating option value. It was suggested that some caution be adopted in the use of the option pricing model in that it is necessary to understand potential problems in the use of such models.

In this paper the topic of option pricing is developed further by examining potential problems in the use of option pricing models. The most critical inputs to the option pricing model are examined and the possibility of model misspecification is discussed. Also some of the more recent alternative models are commented on.

Problems with the Black and Scholes (B & S) Model

When we use the B & S model to value an option, the price we obtain is rarely exactly the same as the actual price of the option. There is a difference between actual and fair value due to supply and demand factors and attitudes and expectations. Also the fair value we calculate may not be the true fair value. There are several reasons why this could occur: (i) we may have used incorrect inputs into the B & S formula, or (ii) the B & S formula may be misspecified. A further confusion arises when investors talk about "improving" estimates of fair value. The author has been surprised that, to many investors, this means obtaining estimates of fair value that are closer to the actual option price. However, what we need is to distinguish between approaches which will give us closer approximations to actual values and those that will give us better estimates of fair value. The first of these corresponds to an 'explanation' approach (discussed earlier in [1]), the latter enables us to make excess profits in the options market.

It is the difference between actual value and fair value that makes it possible for us to trade profitability in options using the formula. Thus it is important to attempt to get the most accurate possible estimate of fair value. The following discussion will focus on obtaining correct inputs to the formula and possible misspecification of the B & S model.

The Inputs to the B & S Model

All the inputs in the B & S formula are known exactly except for the volatility and the interest rate. The main input to the B & S formula that could be wrong is the volatility. [5, p.1] The volatility that is required for input to the model is the volatility of the stock over the life of the option until its expiry. Additionally, B & S assume that the volatility does not change over the life of the option. [5, p.2] The area of volatility measurement in option pricing is worthy of detailed examination in itself. However, space permits us to address it in only a brief manner.

Essentially the task is to forecast the volatility over the period of the option using historical data. We then assume this volatility estimate will remain constant throughout the life of the option.

Option value is quite sensitive to volatility. A superficial look at some of the options literature by an investor may lead to calculations of volatilities which are too inaccurate or misleading. Consider the following advice from the Hewlett-Packard HP-41C Securities Pac Handbook.

Volatility is the annual standard deviation of the return on the underlying stock. There are several ways of estimating it. One method is to use the equation:

\[
\text{Volatility} = \frac{\text{High-Low}}{\frac{1}{2} (\text{High} + \text{Low})}
\]

where the high and low values are those of the stock over a period of time. However, experience has shown that using this method produces values which are too high. Thus, use 6 months highs and lows . . . or dispense with dividing the denominator by 2.

Caution should be adopted in using an estimate based on only two observations. Preferably a daily or at least weekly price series should be used. Ad hoc measures such as halving this volatility are to be avoided. Despite this a number of authors suggest using rough estimates of volatility.

Whilst it is desirable to use daily prices in calculating volatility the period of time to be chosen is far from
obvious. A wide range of periods is suggested in the literature. Francis [19] suggests the use of a five year period. Others use a variety of periods. For example Brown [9] uses 26 weeks, 52 weeks and the period from date of trade to the first, second and third maturity dates. We may well consider that more recent data may influence future volatility more than older data. Thus we may want to give greater weight to more recent data through the use of exponentially weighted moving averages, or some similar technique.

Any quantitative forecast should be modified by sound business judgement and the forecasting of volatility is no exception. We may wish to alter volatility estimates on the basis of an expected event (eg. the result of a Federal election or some other factor leading to a change in stock market price levels). Alternatively, we may wish to use the volatilities implied by actual option prices to alter our historical volatility estimates.

Whatever value is used for volatility, we suggest considerable thought be given to its selection and use. In particular, if an option valuation system provided by a sharebroker or subscriber service is to be used, the basis of the volatility estimates used in that model should be understood.(4)

Another input to the model is the interest rate. As Brown has pointed out the most suitable rate would be “that paid out on a riskless instrument whose maturity corresponds to that of the option”. [9, p.22] Brown uses interest rates from a number of sources including Treasury Notes and yields on non-rebatable Commonwealth Government Securities. [9 p.27] Noti [30 p.24] chooses the buying rate used by the Accepting Houses Association for prime non-bank commercial bills and having the same maturity date as the options. Chiarella and Hughes [13, p.40] use the yield of Treasury Bills, as do Castagna and Matolcsy [11] Brown and Rainbow [10] make further comments on interest rates in the Australian context.

One uncertainty as to which interest rate to use arises from the fact there is a difference between the rate (r1) at which money is borrowed and the rate (r2) at which money is lent. Option buyers are net borrowers and rate r1 is appropriate. Option writers are, in effect, lenders so rate r2 is appropriate.(5) Even if these rates were the same for all investors, it is the after tax return or investment that is important. [42, p.238] Also because “the actual rate of return will fluctuate as a result of the fact that frequent adjustments in the hedge ratio are impractical suggests a rate slightly higher than the risk free interest rate should be used”. [21, p.249] Gastineau states that most sophisticated users of option valuation models add between one and two per cent to the risk free rate. Because of these factors [21, p.296] Gastineau himself adds one and a half per cent to the Treasury Bill rate [21, p.297].

In general the effect of interest rate changes on option price is not really as great as for volatility changes, particularly for options which have an exercise price close to the current stock price.(6)

Possible Misspecification

The possibility that the B & S model is misspecified has been raised by a number of authors including Black himself. Black states “there will be a series of models developed over time that are better than the original Black-Scholes model”. [5, p.2]

A possible major problem with the B & S formula is a misspecification because of the assumed probability distribution. B & S assume that the probability distribution of stock prices is a log-normal distribution. Madansky has shown the variation between the log-normal and an empirically derived distribution [21, pp.324-331]. If the exercise price is close to the current share price the log-normal approximation provides fairly good results. However, if the exercise price and market price vary greatly, the log-normal approximation may lead to a very poor estimate of the fair value of the option [21 p.250].

Merton has argued that the assumption in option pricing that the underlying stock price returns must follow a stochastic process which generates a continuous sample path is a misspecification. This is because it implies that over a short period, the stock price change cannot be very large. [26 p.127] He suggests an option pricing formula which takes into account two types of changes in stock price:

(i) the “normal” variations in price, modelled by a standard geometric Brownian motion, and

(ii) the “abnormal” variations in price due to the arrival of new and important information about the stock that has a more pronounced effect on the price. This is modelled by a jump (Poisson driven) process. [26 pp.127-8] Whilst models such as this may improve the specification of the B & S model they may well be too complicated to use for even very sophisticated investors [21, p.349].

Two further problems in the specification of the B & S model appear because it is assumed that the stock pays no dividend and the effect of taxation is ignored.

Dividends on some stocks can have a significant effect on the valuation of options. [14, p.213] Merton [27] has made an adjustment to the B & S formula such that dividends are assumed to be paid continuously but this obviously does not follow actual dividends which are paid at discrete points in time. Black has extended the B & S model to cover payments of dividends but the new
formula requires a numerical integration procedure that is expensive even on more advanced computers [20, p.183]. However, unpublished research conducted by Black shows a simple adjustment performed by subtracting the present value of the expected dividend payments (over the life of the option) from the exercise price to be a fairly good approximation. [20, p.183] So this would appear to be an adequate adjustment for dividends. Brown and Rainbow [10] and Noti, [30], [32] discuss dividends in the Australian context in greater detail. Castagna and Matolscy [11] test for the effects of dividends in the pricing of options.

Taxation can also affect option value. Noti has pointed out that the effect of taxation can be twofold: (i) taxes on interest payments can lower the effective rate of return on a risk-free hedge, and (ii) the difference between non-taxable gains and taxable income of another nature may lead investors to favour one strategy over another, thus causing market prices to deviate from fair value price. [30, p.10] Scholes [38] looks at the taxation problem for the case where income is taxed at a uniform rate and Noti [31] considers the case where tax is paid on interest received but not on the premium. The general situation regarding taxation of options is discussed in [41, pp.A1-10] and further comments on the effect of taxation on option value appear in [21, pp.127-171].

Alternative Models

Alternative option pricing models have been suggested by others, including Bird and Henfrey [2], Henin & Ryan [22] and Gastineau [21]. Bird and Henfrey adopt an empirical approach based on their work on warrant markets in the U.K. Their approach is based on calculating a predicted option value from actual option prices. The actual option price is then compared to this predicted value. The theoretical basis on which their approach is based is not clear. One interpretation is as follows: the predicted value is based on regression analysis of actual values. This average comprises some values which are higher and some which are lower than the predicted value and could be considered an alternative measure of 'fair value' to that computed by Black and Scholes. If this interpretation is correct we are not convinced this yields a satisfactory measure of fair value.

Henin & Ryan use a random walk model with discrete jumps and hence a discrete probability function governs the changes in price. The distribution obtained is a symmetric price distribution. With such a distribution, if there are downward trends in price, negative share prices can result. This is a dubious departure from current option pricing theory. Both the log-normal distribution suggested by B & S and the empirical distribution derived by Madansky suggest the distribution is a skewed one. The work of Henin & Ryan has been strongly criticised by Thorpe [43] and Gastineau [21, p.343] who suggest this work should not be seriously considered.

The option valuation model that is used by Gastineau is based on the empirical distribution derived by Madansky. This is a departure from the Black & Scholes model which uses a log-normal distribution. The Gastineau formula more closely resembles the valuation formula of Samuelson and Merton [37]. The Gastineau model is described in [21, pp.254-262]. Whilst the work of Gastineau deserves closer attention, there is not sufficient information, to the author's knowledge, to enable option values to be calculated using this model on Australian stocks. It would be of interest to derive empirical distributions for those stocks which are traded on the Australian Options Market to enable option values to be calculated using Gastineau's model.

Concluding Comments

The Black and Scholes option pricing model provides us with a practical method to calculate option value. Unlike many other option pricing models it has the advantage that all the inputs to the model are either known or, in the case of the volatility and the interest rate, can be calculated.

The most important unknown input is the volatility. However, whilst we can use many different ways to evaluate price volatility, there is no way of evaluating volatility based on past price history that can estimate exactly what the volatility will be during the life of the option. The investor who becomes heavily involved in options should pay considerable attention to the problem of forecasting volatility. In particular, if a valuation service is used, the user should be fully aware of the underlying method of computing the volatility variable. The interest rate used in the B & S formula, whilst it is not nearly so sensitive a variable as the volatility, should also be considered.

A number of modifications to the B & S model have been suggested and some of these have already been discussed. In particular, an investor should take into account the effects of dividends and taxation on option value. For options that are traded at prices not close to the current stock price, possible distortion to the fair value result caused by the assumption of a log-normal distribution should be considered.

Whilst several alternative option pricing models have been suggested, none of these is in widespread use. Some of these may, in time, prove superior to the B & S model but detailed empirical testing of option pricing models lag behind the theoretical work in this area.
Finnerty has noted "more effort is required in identifying and evaluating the various market or institutional factors which will lead to a more robust specification of the option valuation model". [17, p.361] With the vast amount of research currently being conducted in this area we can expect considerable further refinement to, and empirical testing of, option pricing models over the next few years.

Footnotes

(2) The references shown in square brackets refer to the bibliography given in Payne (1) above. The references are excluded because of space considerations. For the Black and Scholes reference see [3].
(3) See, for example Noddings and Zazove [29].
(6) The effect is greater for in-the-money options. For a further discussion on interest rates see Noti [30, pp.5-81], Fuller and Wang [18, pp.18-19], Black [5, p.6] and Thorpe [42, pp.237-240].