Volatility may not always be what it seems. In the interest-rate options market, the differences between price volatility and yield volatility can be challenging.

Conversion Formula

There is a quick and easy conversion formula between the two volatility measurements for interest rate options:

\[ V_p = -V_y \cdot Y/P \cdot dP/dY \]  

where \( V_p \) is the price volatility, \( V_y \) is the yield volatility,

\( Y \) is the yield of the underlying security,

\( P \) is the price of the underlying security, and

\( dP/dY \) is the first derivative of price \( P \) with respect to yield \( Y \) of the underlying security.

After simple manipulation the formula becomes:

\[ V_p/V_y = -Y/P \cdot dP/dY \]  

This expression tells us the ratio of price volatility against yield volatility at different yield levels and corresponding prices for an interest-rate security. One can therefore investigate the relationship between \( V_p \) and \( V_y \) for different levels of prices or yields. Mathematically, a simple first derivative with respect to \( P \) (price) of this expression will show this relationship.

Relationship between the two volatility measurements

Mathematically, the first derivative with respect to interest-rate security price of Expression (2) is the following:

\[ Y/P^2 \cdot dP/dY \]  

which happens to be always negative. This means that as the price of security goes up, the ratio of yield volatility to price volatility is negative.

Jeremy Lin is with Group Treasury, Commonwealth Bank of Australia.
price volatility to yield volatility decreases. That is, a higher security price is translated into a higher yield volatility for a given level of price volatility. Or, for a given yield volatility level, the corresponding price volatility decreases as security price increases.

Graphs 1 and 2 show the price/yield volatility ratio for the XYB and CGS 7/2000 stock.

Other characteristics of \( Vp/Vy \)

(a) This ratio is constant at a given yield for the underlying security.

(b) At a constant price, the shorter the term to maturity of an instrument, the lower the ratio, or the discrepancy between yield volatility and price volatility is larger. \( dP/dY \) in Expression (2) can be used as a proxy for duration.

As the maturity draws close, the duration is shortened, therefore \( Vp/Vy \) is smaller.

For example, bill options have a price volatility of about 0.25 per cent with a corresponding yield volatility of 10 to 12 per cent; this compares with 7.5 per cent price volatility and 12 per cent yield volatility for the 10-year bond futures contract.

(c) This differential between the two volatility measurements for securities of different maturities is more pronounced in a higher interest-rate environment.

Practical implication of the ratio \( Vp/Vy \)

Some practitioners in Australia use yield volatility for option trading to reflect their familiarity with the yield-pricing convention for interest-rate instruments.

Naturally, records for implied volatility and historical volatility are generally in terms of yield volatility. Any statistical analyses are therefore in the yield volatility concept. This leads to some interesting observations.

One simplistic method to determine the relative level of implied volatility is the use of the average and standard deviation for a historical sample of volatility. Roughly, 68 per cent of all observations fall within one standard deviation from the average of the sample population. For example, if the average is 10 per cent and the standard deviation is 2.5 per cent for a sample of implied volatility, 68 per cent of the observations will fall between 7.5 per cent and 12.5 per cent. Therefore, if history repeats itself, future observations should more than likely fall within this band. This can be used as a simple guide for trading volatility.

However, this trading guide alone is not accurate because a substantial change in one direction of the price of the underlying interest-rate security over a short period would distort this analysis.

Take the 10-year bond contract traded on the SFE as an example. The nominal dollar price for this contract rose from a low of $90,000 to about $109,000 per contract over a six-month period from November 1990 to May 1991. Graph 3 shows the steep increase in the contract price for this period, and tends to be along one direction. On the other hand, the price change for 16 months before this period had been over a steady band.

Graphs 4 and 5 show historical implied price and yield volatilities from the beginning of July 1989 to end of May 1991. The central straight line is the average for the period, the upper straight line is the sum of the average plus one standard deviation, and the lower straight line is the average less one standard deviation. Graphs 6 to 9 are for the three-year and bank-bill contracts. The implied price volatility figures for the futures contracts are from ICCI.

Graph 4 shows the price volatility

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