Hedging and the role of the exploding lattice

Hedging is an inexact science, but it is better to hedge imperfectly than not to hedge at all, writes Ian Bell.

Technological advances have led to a hedging and pricing model applicable to capital guarantees.

Life offices have been writing put options against diversified equity, property and bond portfolios over the past five years and not hedging their positions. These options are embedded in capital-guaranteed investment account contracts.

However, recent regulation now forces those life offices with a low reserves position to either "portfolio insure" or to shift to a "cash plus calls" strategy.

This has implications for institutional responses if there is any further substantial fall in the All-Ordinaries index. Further falls in property values are a separate issue about which the offices can do nothing, except perhaps to emphasise further the use of the sharemarket as an outlet valve for their overall liquidity needs.

This article shows how the risks undertaken by these life offices can be modelled using an extension of the framework of the Cox-Ross-Rubinstein binomial option-pricing model. This type of model can enable conclusions to be reached about the adequacy of an office's solvency reserves.

It can also indicate the hedging program that would be needed to preserve the office's solvency or, alternatively, the changed asset allocation necessary to minimise the risks incurred through option exercise.

Origins

Life offices traditionally provided mainly whole-of-life and endowment assurance contracts, but from the 1970s consumer pressures led to a trend towards "unbundling" — separating the contractual definitions of life insurance cover, investment and expense loading elements. Once this was done, policies could be redesigned by combining these elements in different ways.

In the 1980s, the growth of consumerism led to companies concentrating on different approaches. Some specialised in temporary insurance which provided mainly a yearly renewable life cover and had few investment implications. Other companies sought to compete mainly for the savings or investment element.

A key consideration here was the perceived threat of competition from the banking sector, either through bank savings products or through banks entering the life insurance sector with the advantage of an established distribution network.

The capital-guaranteed investment account product emerged in this period. Perhaps not surprisingly, some forms of this business bear an uncanny resemblance to term-deposit accounts, when viewed from the policyholder's perspective.

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The disclosure provided a watershed. Consequently, some offices altered their outflows began as a response to plummeting investment account policy, noting no exceptions for the life office's investment pool backing this class of business. Charges are deducted for administration and for tax.

The critical feature of the policy is the investment guarantee provided by the office. Typically, this would relate to the full account balance, including all interest credits to date, although fees and other charges could be deducted by the office on early surrender of the policy.

Often the policies were marketed by comparison with bank accounts, showing that the investment pool (typically a balanced portfolio — i.e., diversified across sectors) historically recorded higher returns. Too frequently this marketing implied that early withdrawal was an exceptional event from the office's perspective, and that the account surrender value would be readily available at any time.

The offices had large free reserves, so that guaranteeing these benefits, regardless of current market conditions, was not perceived (except by some offices notably) to be a sizeable risk.

The volume of capital-guaranteed business of the life offices grew substantially after the 1987 stockmarket crash, as investors sought to protect their remaining assets.

The options analogy

A paper presented to the 1989 Convention of the Institute of Actuaries of Australia was the first to identify the offices' capital guarantees as de-facto written (or sold) put options. The disclosure provided a watershed for capital-guaranteed business; subsequently, some offices altered their policy terms or even stopped writing new business of this kind. In 1992, outflows began as a response to plummeting crediting rates.

A guarantee of a capital sum of $X on an investment account policy, no matter how $X was defined, was seen to be the equivalent of a put option granted to the policyholder with $X as the exercise price of the option. While not all policyholders could be expected to apply economic rationalism to the choice implicit in this option, the growth of the investment advisory profession has certainly increased the likelihood of a purely financially-motivated election were the option to fall “in the money”.

The importance of this implied put option depends directly on the investment strategies followed by the office for the assets backing the policy. While information is not readily available to investors on the supporting asset mix, there is a theoretical “asset share” that the policyholder has in the relevant statutory fund. This asset share is made up of equities, of direct and listed property investments, of bonds and of short-term securities.

The asset share is unallocated, but can be readily determined by the office concerned. Under usual conditions, the asset share would be worth more than $X, but if the office follows risky investment strategies, or if there is a substantial fall in either equity or property market values, the capital guarantee incurs a top-up cost for the office, in the same way as an option.

Suppose, for the sake of simplicity, that the office always kept a constant proportion of the market value of its investment portfolio allocated to the different sectors. An example might be a 40:20:30:10 mix for equities, property, and cash. Suppose also that the office's investments in each sector are representative of the market in each sector, i.e., they achieve sector index performance. If this is the case, the office has effectively written a basket put option on a 4:2:3 mix of the risky sector indices (cash being regarded as risk-free).

If we assume (as in the usual case) that the sector proportions change over time, then we simply have a more complicated put option with changing basket proportions. Further, if we assume that property value risks cannot be hedged, then property investments will need to be supported by free reserves, thereby reducing the reserves available to support the guarantee. In this event, a two-sector basket put option remains, but with a higher effective exercise price, thereby increasing the risks to the office.

The model

These basket put options can be modelled using modern option-pricing theory. However, some specific points seem unique to the life office capital-guarantee case, and mean that standard Black-Scholes modelling techniques are insufficient.

This has led to the use of a discrete lattice framework based upon the style of the Cox-Ross-Rubinstein binomial model, but extended significantly beyond it. There are many variations in product design and one needs the flexibility to model many special features. To do this, one needs the ability to look inside what could be called the “Black-box” of the pricing mechanism for various purposes.

Major differences from Black-Scholes

- The typical life office guarantee results in a changing, and generally increasing, exercise price over the duration of the policy. This starts with the policy surrender value, and ends with the policy maturity value.
- There are many expense and account-charge effects which need to be modelled and these are mathematically like the dividend effects in an equity-warrant pricing model.
- Annual premium effects are like a negative dividend effect since the asset share is being increased whenever a premium is credited to the account.
- Where premiums are paid into the account, some variation occurs in the exercise price, but not necessarily on a one-for-one basis.
- There are regions of inactivity in early exercise, where policyholder reaction lags, and the degree of tolerance to temporary emergence of intrinsic value, should be taken into account.
- Many of the discrete cashflow effects result in increasing complexity. These negate many of the assumptions of the Black-Scholes model. Recurrent premium effects also rule out use of a simple binomial model. Further, some crediting-rate formulae give
path-dependent effects similar to the Asian options in the currency area, thereby requiring further detailed modelling treatment.

To go into the detail of the above differences from Black-Scholes would be a lengthy academic exercise. However, it can be said that by using a generalised form of binomial lattice, the unique features of a capital-guaranteed contract can be suitably modelled.

**The computer algorithm**

We have found that modern computer programming techniques give a means for modelling both an exploded and a connecting lattice in the same basic set of algorithms. This is achieved by something analogous to a recursive formula. Consider Figures 1 and 2. The lattice relates to movements in the value of the assets behind the option. At each node (branch) the value can go up or down in accordance with a predetermined volatility parameter.

Figure 1 shows the simple pattern of the binomial tree. Because an up-move cancels a previous down-move, and vice versa, the number of end-nodes is much smaller than the number of individual paths along the tree when we use 100 or so steps, as is usually the case.

In fact, as the number of steps becomes larger, these weightings of nodes will start to approximate to the Normal probability distribution. Moreover, if, like most option-pricing formula authors, we set the parameters for up and down moves in a particular way, we can approximate to the Lognormal distribution at the end-nodes.

What we find is that the option values obtained are not path-dependent. That is, the value represented by a given end-node does not depend on which path the tree followed to get there. This is a key assumption underlyng both the Black-Scholes model and the Cox-Ross-Rubinstein binomial model.

Figure 2, on the other hand, if continued step-by-step for the rest of the lattice, would not give a standard probability distribution at the end-nodes. We can still have such a distribution underlying the price movements in the absence of the displacement effect. However, the displacements destroy the connections in the lattice and give what we call an exploded lattice.

An exploded lattice can accommodate certain path-dependent features — for instance, if the exercise price incorporates an average of outcomes at different points along the way, such as when crediting rates are "smoothed" over time.

The difficulty is that the structure in Figure 2, when extended for a large number of steps, is not only substantially more complex; it also takes up far more computer time. A 100-step model of the type in Figure 1 involves 101 end-nodes, but for the fully exploded version in Figure 2 the number of paths and end-nodes would both equal $(2)^{100}$ which is a very large number, of the order of $10^{30}$. The exploded case would be impractical in terms of run-time on a PC even for a small number of steps.

We get around this by using more powerful programming tools. We also take things further by going through the exploded step logic only when there actually is an explosion of nodes. This means we get optimal speed if we
are doing a simple binomial tree, and also use the minimum possible run-time if there are some exploding steps and some connecting steps along the lattice.

Since we are modelling discrete events in time, the latter mixed style of lattice is the more usual kind. Figure 3 shows this form of structure.

There are other effects that are not explained here, such as differing step sizes, varying volatility from step to step, and other approximations to reality, which can all be accommodated within this general framework. Limits to accuracy are governed more by computer run-time considerations than by the model's methodology for representing reality.

Uses for the model

As with any area of risk, different functions must be performed in which sophisticated mathematical treatment can provide a significant degree of added value. The use of a suitable option-pricing model in the life assurance risk-management area is no exception. The parallels with treasury risk-management functions are shown in the table below.

The first attempts were made in 1989 to introduce option-pricing techniques into the area of life office capital guarantees. Stochastic simulation techniques have also come into use, aiming at much the same result.

In 1991 the author and an AMP actuary, Peter Hodgett, submitted proposals to the Biennial Convention of the Institute of Actuaries with respect to capital adequacy and solvency considerations (being mainly function 2 in the table).

Since that time a number of offices have considered, and some have implemented, hedging techniques for coping with reserves exposure through capital-guaranteed business. Others, believing their reserves are sufficient to withstand further falls in the share and property markets, obviously do not yet regard hedging as a matter of great importance.

The model we describe above can be used in all three risk-management functions. It was originally devised for the purposes of solvency analysis work, but the nature of this means that it is naturally applicable to setting policy terms and conditions (ie, premium rates, being function 1 in the table).

Closing thoughts

We prefer the openness and visibility of a discrete lattice model, where the inner workings can be easily displayed on a computer screen, to the hidden approach of many option-pricing models. The approach we have taken enables the model to be physically audited (usually by spot-check methods, due to its size) and for users to educate themselves on the assumptions and their implications or effects.

Hedging is an inexact science, but with option positions the development of enhanced skills and technology will make a difference. "Better to hedge imperfectly than not to hedge at all" might be a useful adage. We would add that advances in technology and the quality of information will be reflected in improved awareness and effectiveness.

Options technology is continually evolving whether in the area of over-the-counter bond options, exotic FX options, or elsewhere. This article has highlighted another developing area, that of investment guarantees issued by life assurance companies.

We have proposed a discrete lattice options pricing and hedging model that has been used successfully in pricing specific forms of capital guarantee. The model discloses its inner workings quite openly to the user and can be easily audited and examined.

NOTES

1 Insurance and Superannuation Commission, Circular No. 273, under the Regulations to the Australian Life Insurance Act, 1945.
