WHERE TO INVEST?
ASK THE COMPUTER
OPTIMISATION: A NEW INVESTMENT TOOL

Decision-making in the capital markets has never been more difficult. There are growing pressures from competition, market complexity, trading volatility and the need for vigilance over security. Additionally, there is a heightened focus on management accountability. In short, managers are under increasing pressure to get things right: to “optimise” their investment decisions.

Australia has been in the forefront in introducing optimisation to the capital markets through personal computers.

The mathematical techniques of optimisation grew out of the field of operations research which was developed in Second World War defence strategies. Operations research has had many applications, such as in manufacturing, inventory control and airline schedules and, in more recent years, the capital markets.

While optimisation for the capital markets was originally restricted to mainframes, its current use on PCs is a significant breakthrough. Some limited forms of optimisation are now becoming available on spreadsheet software.

There are currently two main categories of financial applications for optimisation:

- optimisation in asset allocation for institutional fund managers.
- optimisation in asset allocation for institutional fund managers.

More applications are being developed, especially in securitisation and annuities management. However, the intention of this article is to introduce the topic of optimisation and explain how it is currently being applied in the capital markets.

Bond and debt portfolio optimisation

For bond and debt portfolios, optimisation can involve the following three main activities:

- optimising investment selection against a liability cashflow profile;
- optimising investment selection against a benchmark asset portfolio;
- strategic decision-making in relation to predetermined planning horizons.

While optimisation theoretically can be achieved manually, the time involved makes using a computer essential. The way the software works to achieve optimisation for investment portfolios is that it builds in a number of constraints which can be applied simultaneously. For a bond or debt portfolio optimisation, major constraints

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which can be established include:

Duration (weighted term) or modified duration: measuring the primary, or first order, impact of interest-rate risks.

Convexity: the next level, or second order, of interest-rate risks.

Dispersion of cashflows ($M^2$): looks at the spreading of cashflows.

Market price/rate of return: placed in the context of other constraints and compared with market universe.

Cashflow: allows specified maximum and minimum limits over specified time periods.

Credit: defines limits on exposure to different grades of credit.

Sector: establishes sector weightings in conjunction with credit constraints (eg, government vs semi-government vs corporate).

Term: applied to terms of securities and allows for yield-curve variations and risk-spreading.

Face value: allows for constraints globally and for individual securities.

Parcel size/round lots: activates integer programming which supplies practical whole-parcel size solutions, allowing for different parcel sizes.

**Technical background**

The technical background to building in these constraints to optimisation involves a variety of mathematical techniques.

These include linear programming, integer programming, quadratic programming, goal programming and dynamic programming.

Each has a different role to play and is appropriate for different circumstances.

Most bond portfolio optimisation problems require only linear and integer programming methods.

Since linear programming is relatively easy to explain when working with a limited number of variables and constraints, we will present a simplified textbook example to communicate the basic elements of optimisation.

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### Example: Playsafe Insurance Company

The Playsafe Insurance Company has idle funds available for investment, but it is subject to government regulations:

- No more than 80 per cent of all investments may be long-term investments;
- No more than 40 per cent of all investments may be invested at short-term; and
- The ratio of long-term investments to short-term investments must not exceed three-to-one.

Conditions are such that long-term investments yield 15 per cent annually, while the annual yield for short-term investments is 10 per cent. Playsafe has $20 million to invest.

The problem: to determine the amounts which should be invested in long-term and short-term investments, subject to satisfaction of the regulations, with the objective of maximising the annual (weighted average) yield.

The procedure: this problem can be expressed as a very simple linear programming problem, which is two-dimensional. The key variables are:

- $L =$ long-term investments proportion;
- $S =$ short-term investments proportion.

The solution is subject to four linear constraints, and the objective function (or goal) is linear. Because the problem is only two-dimensional, it can be solved by drawing a simple two-dimensional graph and examining key points on the graph.

We have already defined our two variables as $L$ and $S$. We can also easily write out the main constraints, represented by the government regulations, as follows:

- $L \leq 0.8 \times 20$ (1)
- $S \leq 0.4 \times 20$ (2)
- $L \leq 3 \times S$ (3)

These are all simple linear inequalities. Because of the amount of available funds, we have another constraint:

- $L + S = 20$ (4)

which is a simple linear equation.

Finally, to specify our objective function, we define a parameter $Z$ as the variable to optimise. In this case, we want to maximise $Z$ where $Z$ is defined by a linear function of interest earnings, namely:

$$Z = 0.15 \times L + 0.10 \times S$$

The solution can now be obtained as shown in the simple two-dimensional Graph 1.

Our solution space, or feasible
region, would be the area within Lines 1, 2, 3 and 4 on the graph, except that we can see that one of our earlier-mentioned constraints (No. 4) is an equality rather than an inequality. Therefore the feasible solutions are reduced to those on the straight line between points (a) and (b).

It can be shown that the optimal solution will be at the extremes of this line.

At point (a), value of \( Z = 0.15 \times 15 + 0.10 \times 5 \)
\[ = 2.75 \]
At point (b), value of \( Z = 0.15 \times 12 + 0.10 \times 8 \)
\[ = 2.6 \]

Therefore, point (a) is optimal, given the constraints specified.

Knowing that the optimal solution must lie at an intersection point of lines which define the feasible region (the principle on which the simplex algorithm relies), in this simple example we can inspect the points one by one. Alternatively, the objective function is a sloped line that can be moved across the graph in one constant direction, progressively changing the value of L and S, until the farthest extremity of the solution space is reached and the optimal solution found.

Some theory: It is worth looking at some of the important features of a linear programming (LP) model generally:

- Divisibility: variables must be infinitely divisible but, if not, then more advanced integer programming is needed.
- Non-negativity: all variables must be non-negative, or capable of being expressed non-negatively.
- Linearity: all relationships between variables must be linear, which implies both proportionality of contributions and additivity.
- Objective function: expresses the goal as a linear equation of relevant variables and operates to find the optimal solution from a range of feasible solutions.
- Simplex method: application of an algorithm (a set of logical and mathematical operations performed in a specific sequence) which, sequentially, workstowards a better solution in a time-efficient manner. This is essentially a computationally efficient search routine that relies on a principle that the optimal solution will be at an extremity of the solution space or feasible region.

### Practical application

This Playsafe example is simple because it is a two-dimensional problem. Optimisation in the real world is far more complex. Practical business problems can involve hundreds of parameters (and hence hundreds of dimensions) and hundreds of constraints, many of which may not be linear. These problems are impossible to solve graphically. Software in use today in the capital markets has been developed to meet these conditions. The accompanying samples of computer screens, from one of the systems currently available, show some constraints.

<table>
<thead>
<tr>
<th>FinancialWare Security Manager v2.518</th>
<th>24/04/1992</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constraints</strong></td>
<td><strong>Mode:</strong></td>
</tr>
<tr>
<td><strong>OBJECTIVE</strong></td>
<td>Change</td>
</tr>
<tr>
<td>Minimise Market Price</td>
<td>W</td>
</tr>
<tr>
<td><strong>CONSTRAINTS</strong></td>
<td></td>
</tr>
<tr>
<td>Total Face Value</td>
<td>between</td>
</tr>
<tr>
<td>500,000,000 and 550,000,000</td>
<td>550,000,000</td>
</tr>
<tr>
<td>Market Price</td>
<td>Objective</td>
</tr>
<tr>
<td>Modified Duration</td>
<td>equal to</td>
</tr>
<tr>
<td>Convexity</td>
<td>greater than</td>
</tr>
<tr>
<td>M^2</td>
<td>between</td>
</tr>
<tr>
<td>4.000 and 5.000</td>
<td></td>
</tr>
<tr>
<td>Scaling</td>
<td>Yes</td>
</tr>
<tr>
<td>Scaling variable</td>
<td>Total Face Value</td>
</tr>
<tr>
<td>Target</td>
<td>485,000,000</td>
</tr>
</tbody>
</table>

### FINANCIALWARE Term Constraints

<table>
<thead>
<tr>
<th>Security Manager v2.518</th>
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<tbody>
<tr>
<td><strong>Term Constraints</strong></td>
<td><strong>Mode:</strong></td>
</tr>
<tr>
<td><strong>Term Range</strong></td>
<td><strong>Minimum %</strong></td>
</tr>
<tr>
<td>1.0 yrs to 1.0 yrs</td>
<td>5.00</td>
</tr>
<tr>
<td>1.0 yrs to 3.0 yrs</td>
<td>20.00</td>
</tr>
<tr>
<td>3.0 yrs to 5.0 yrs</td>
<td>20.00</td>
</tr>
<tr>
<td>5.0 yrs to 10.0 yrs</td>
<td>35.00</td>
</tr>
<tr>
<td>10.0 yrs to 99.9 yrs</td>
<td>4.00</td>
</tr>
</tbody>
</table>

Calculate Target Values: Select

**Licensed User**: FINANCIALWARE

**SYDNEY**

JASSA JUNE 1992
miser Market Price”. This is the same as maximising yield. A number of other constraints are also available.

- Constraints: these are limits placed on the statistical characteristics on the replicating portfolio.
- Scaling: this brings the market-price related characteristics of the index into the same ballpark as the replicating portfolio.
- Extra constraints: more specific constraints may also be placed on the model.

The next screen (2) shows some more specific constraints on the terms of the securities in the replicating portfolio. The optimiser is being forced to select securities across the yield curve in the defined term ranges. The minimum and maximum percentages are the specified limits. The proportions of maturities in the target portfolio are shown as an aid to determining these constraints.

Two constraints have been placed on the face values of the replicating portfolio:
- maximum face value of any one security of $50 million, to keep the parcel size within realistic limits.
- minimum parcel size of $1,000,000—this constraint activates the integer programming function and produces realistic market parcels.

Screen 3 compares the characteristics of the index with its replicating portfolio. The statistics for the index are shown in the column INDEX, while the replicating portfolio is shown as UNIVERSE. The difference column is simply (INDEX-UNIVERSE).

Significantly, there has been an average yield pick-up of about 11 basis points. This is the equivalent of more than $1,000 per million per annum.

An important point to emphasise about the practical application of optimisation is that it is not a black box which magically produces the right answer, nor is it programmed trading. To be effective it requires management input and informed interpretation of data.

In practical terms, there may not always be a single best answer and over time the specification of the problem may change. However, the basic mathematical techniques of operations research, and their computer programming application to the capital markets, are universal. Optimisation applies in all markets around the world, with only minor allowances for differences in detail for some markets.

### Liability management

While bond and debt portfolios are often considered mirror images of each other, there are a number of differences that need to be considered in optimising the management of liabilities. Among the key differences are the cost of holding debt versus return on assets, horizons of performance measurement, the universe of securities, credit issues, accounting practices and definitions of risk.

However, allowing for these differences and the need for managers themselves to make their own yield-curve forecasts, optimisation is currently being applied in liability management by several semigovernment borrowing authorities. Some of the ways it is being used are:
- Creating practical portfolios that replicate the risk statistics of idealised benchmark portfolios. The performance of the liability manager can then be measured against this practical benchmark.
- Determining areas of the yield curve where it is comparatively cheap to issue.
- As a decision support tool translating interest rate views into practical market trades.

### Portfolio immunisation

A useful application of optimisation for bond and debt portfolios is in the immunisation of these portfolios against interest rate risk. The starting point for any immunisation strategy is to adopt principles developed in the early 1950s by the UK actuary F.M. Redington.

Redington’s concept, once classified as an ideal by life assurance companies worldwide, was that an immunised portfolio should have the mean term of the value of the assets equal to the mean term of the value of the liabilities and must keep the spread of the assets about the mean term greater than the spread of the liabilities.

In the terminology used in bond portfolio analysis, Redington immunisation would apply the following two rules:
- that the modified duration of assets was equal to the modified
duration of liabilities; and
that the convexity of assets is
greater than the convexity of li-
abilities.

The Redington immunisation
model assumes that the yield curve
moves in parallel shifts. In reality,
it is rare to see a parallel shift of the
whole yield curve. The curve can
twist or the short end can move
while the long end remains the same
and vice versa. The most damag-
ing shift in yields would usually be
from twists in the curve. To suc-
ceed, immunisation must address
these issues.

One solution to this problem is
to subdivide the yield curve into
small intervals and immunise each
sub-interval. If the sub-intervals
are small enough, the yield curve
movements will be approximately
parallel for each interval for most
yield curve movements.

By applying immunisation to
sub-intervals or segments of the
yield curve, it is possible to get
around the major shortcoming of
the original model presented by
Redington.

This segmented immunisation
can be easily implemented using
linear programming optimisation
techniques. The required solution
is to match the market value of
assets and liabilities, match the
modified duration of assets and
liabilities and keep the convexity
of assets greater than that of li-
abilities.

Examples of other constraints
which can easily be added to the
flexible linear programming model
are:
- specifying minimum amounts for
  selection of “hot stocks”, i.e., to get
  a result with ample liquidity attributes;
- specifying minimum parcel sizes;
- limiting credit quality by both
  rating and issuer.

The net result, if this process is
followed thoroughly, is that for a
given shift in the yield curve (up or
down), a portfolio’s assets will per-
form well enough to more than off-
set any impact on the market value
of liabilities. This effect is similar
to that shown in Graph 2.

Note that in the example the
portfolios are each worth about
$200 million. There are 16 securi-
ties in the TARGET portfolio and
6 in the REPLICAT portfolio. In-
teger programming was used.

Across a 2 per cent yield curve
shift, the two portfolios differ in
value by about $4,000.

While this may sound like a case
of “Heads I win, tails I win”, the
practice is not quite as easy as the
theory. This is because the
Redington theory only deals with
an instantaneous change in yields.
The theory implies a constant re-
balancing of the portfolio with con-
tinuous trading, to provide dy-
namic adjustment with each altera-
tion in the yield curve.

Since these constant adjust-
ments are not made in practice, an
element of judgment must be in-
troduced to make the theory work
in practical situations. This is a
classic case of advanced decision-
support computing tools.

Additionally, software develop-
ments since Redington’s work
means that portfolio optimisation
need not be confined to the classic
Redington approach. It is possible
to introduce interest-rate views and
do a holding-period optimisation,
or introduce probability weightings
to projected yield curve scenarios
and optimise according to rolling
horizons.

Asset allocation

Many fund managers would be
familiar with a variety of optimisation software. Most
optimisers are used in equity index
replication to minimise index-
tracking error. At least one Aus-
tralian investment bank is using
optimising software to support its
bond-sales activity. Other uses of
operations research techniques are
being developed by other market
participants.

An emerging use of optimisation
Continued on page 29
discussed—but without the aid of diagrams to help the reader’s understanding.

This absence of diagrammatic content is one of the book’s major shortcomings.

In dealing with metal price forecasting, the author explains the factors affecting commodity prices and the differences between precious, base and industrial commodities, but the chapter lacks instruction on applying these factors in a logical framework to make competent forecasts. Only one giveaway comment could be applied to a forecast: “. . . gold’s apparent five-year cycle should not be ignored when contemplating its future price . . .”; but little supporting evidence for this is provided.

Discussing mine financing, cashflow analysis and net present value (NPV) concepts, the author asserts that as a mine is a wasting asset, over the long term an investor will receive only the dividend stream.

The NPV of the dividend stream should therefore be the basis for any long-term valuation of a mining company.

This is applicable in the South African context, where gold-mining companies distribute their cashflows to shareholders, but in the Australian context it would lead to some interesting valuations.

This point is clarified in a chapter on mining company valuations, which points out that NPV of dividends is not truly applicable to non-South African stocks. Profitability and dividends payable are recommended as the main considerations in the valuation of gold shares, wherever situated, although the applicable methodology is not clearly explained.

The final section of the book deals with trading in mining shares and charting, and reviews issues such as timing, quality of companies and management, and construction of portfolios. Various markets and their functions are explained, although there is an error in a reference to crossings on the Australian SEATS system.

Little is said about charting techniques. The remainder of the book discusses the application of implied value for South African investment houses, price-to-book-value concepts and the capital asset pricing model.

Perhaps the best piece of advice in the book appears on the final page, where the author advises his readers not to be greedy.

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OPTIMISATION

software is in asset allocation, where the optimiser assists in balancing investments between equities, fixed-interest and cash sectors, and between futures, options and physical instruments.

Managers may be called on to explain the reasoning behind their forecasts, which are often made by weighing up the results and probabilities of several scenarios. Probability-weighted horizon analysis provides a technique for achieving measurement and sensitivity analysis of these variables.

Optimisation software can be used to select ideal weightings in the equity, cash and bond sectors, together with the appropriate degree of futures and options exposure in each of the sectors.

An emerging area for the application of optimisation is in the management by life companies of annuity products. The annuities market has grown significantly in recent years.

Since business success in this area depends on tight management of assets and liabilities, optimisation will be increasingly used as a management tool.

The developing area of securitisation is another fertile field for the application of optimisation. The setting of market and management constraints in the selection of replacement or substitute stock portfolios for large numbers of small parcels of illiquid stocks is a perfect application of the optimisation methodology.

VIEWPOINT

MIC MONEY

Sir,

I would like to add to comments by Chris Golis in the March 1992 issue of JASSA. In the order of his comments:

1. There was no suggestion in my earlier article (JASSA, December 1991) that “holders of MIC licences would get more than $10 million”. There was comment, based on US experience, that $10 million was a minimum for an effective fund. Chris supports this with “all efforts then were directed to obtaining new MIC allocations . . . and shutting out new applicants”.

2. My suggestion was that the MIC scheme could just as easily have offered a 25% per annum deduction with no clawback. Investors would have had a smaller tax deduction but liquidity immediately on listing.

3. Contributing shares would not have created a nightmare. Administration costs would have been similar but there would have been certainty in future fundraisings. I ask why raising $320 million is unrealistic. What has the raising of equity got to do with the “leverage calculations of the 1980s entrepreneurs”?

4. Listing of MICs was a waste of time and money. The fact that the promise of liquidity was necessary in order to raise funds does not alter this.

5. Chris is here agreeing that “shareholder liquidity did not exist”.

6. My comment was that there should be “minimal constraints with maximum freedom”. There was a list of both constraints and freedoms. With which does Chris agree or not agree? What others would he suggest?

We have to live with a recently announced government-backed scheme. Comment on its details and views on how well it will work should be encouraged.

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University of Newcastle