A major limitation of the classical implementation of mean-variance analysis is that it ignores the effect of measurement error on optimal portfolio allocations. In this article Dave Allen and Richard Sugianto report on a study confirming that strategies which control for estimation risk perform better than those which do not. In particular, the minimum variance portfolio, which ignores information on the ex-post mean returns, outperformed all other strategies in the sample period. Even more startling was the fact that it outperformed the Australian All-Ordinaries Index 80 per cent of the time on the basis of Sharpe performance measures.

One reason why Markowitz's portfolio optimisation model has not been used extensively, despite its theoretical reputation, concerns the estimation problems associated with the inputs. Although the ex-post Markowitz mean variance portfolio is generally accepted as the efficient portfolio, the choice of an efficient Markowitz portfolio in the ex-ante case is problematic.

The ex-post portfolio is a term used for a portfolio constructed using perfect hindsight to calculate the inputs. That is to say, the actual returns, variances and correlations of each asset are known when the portfolio is formed. By definition, the ex-post portfolio is always mean-variance efficient. By comparison, the ex-ante portfolio is one where the inputs are not known and will therefore need to be estimated. Only in the extreme case where the inputs are estimated with 100 per cent accuracy will the ex-ante portfolio be mean-variance efficient.

A serious difficulty associated with estimation risk is the instability of the weights attached to the optimal portfolio. Variations in expected returns can change the weights quite dramatically. We report the results of the application of a series of portfolio analyses which adjust for estimation risk. This involves using a class of shrinkage estimators proposed by Stein (1955). The use of these estimators has been applied in studies of international portfolio diversification strategies from a US context by Jorion (1985, 1986), Eun and Resnick (1988, 1992) and in an Australian study by Izan, Jalleh and Ong (1991).

We report the results of tests of the ex-ante accuracy of mean variance portfolio theory using past data as the inputs. We examine whether there are any portfolio selection strategies which can outperform the market. The analysis is applied to the Australian industry groups and two portfolio benchmarks are used — the Equally Weighted Naive Portfolio (where equal weight is given to each index) and the Australian All-Ordinaries Index.

Development of the empirical evidence about the ex-ante performance of portfolios leads to interest in various other strategies, especially given the unsatisfactory ex-ante results of the classical tangency portfolio, which Izan, Jalleh and Ong (1991) termed the Certainty Equivalent Tangency Portfolio. Attention has now been turned to other strategies, such as the Minimum Variance Portfolio, which may be less subject to estimation errors.

Another alternative is to use Stein's shrinkage estimators. The basic idea behind these estimators is to shrink the index sample mean towards a common value which is less likely to be affected by extreme observations than the in-
ferred to as the estimation period (approximately two years). The portfolio assumption that an investor has prior knowledge of the historical samples of optimal portfolio weights. This is referred to as the estimation period (approximately two years). The portfolio

The data used for the risk-free rate is the yield on 26-week Treasury notes for the period of 126 trading days. Jorion (1985) noted that the weights of the Minimum Variance Portfolio depend only on the sample covariance matrix while, on the other hand, the Classical Tangent Portfolio also relies on sample means. Since the sample means are said to be imprecisely estimated, more emphasis should be placed on the Minimum Variance Portfolio.

The results of a number of approaches based on different portfolio strategies are presented. The Bayes-Stein estimator is used as one of the approaches to the estimation of the inputs. We examine whether this approach dominates the classical mean variance approach in the ex-ante period, and whether the Markowitz approach in general, the Minimum Variance Portfolio, the Certainty Equivalent Tangency Portfolio and the Bayes-Stein Tangency Portfolio, dominates both the Equally Weighted Naive Portfolio and passive diversification via the All-Ordinaries Index.

The study used the daily data from the 23 accumulation indices and the accumulation All-Ordinaries Index published by the Australian Stock Exchange. The sample covers the period from 4 January 1988 to 24 June 1992 and used daily continuously compounded returns from the accumulation indices.

The Australian accumulation All-Ordinaries Index is used as one of the benchmark portfolios (along with an equally weighted portfolio of the 23 sub-indices). The All-Ordinaries Index sample consists of all the companies covered by the 23 sub-indices. The data used for the risk-free rate is the yield on 26-week Treasury notes at the beginning of each holding period of 126 trading days.

The analysis proceeds on the assumption that an investor has prior knowledge of the historical samples of 500 daily returns on the 23 sub-indices which he uses to estimate the optimal portfolio weights. This is referred to as the estimation period (approximately two years). The portfolio purchased and held for the holding period; that is, for 126 trading days subsequent to the estimation period (approximately six months). The windows for the estimation period and the holding period are then shifted forward every 21 trading days (approximately one month). This will leave 25 out-of-sample holding periods that can be used to compare the performance of various portfolio strategies on an ex-ante basis.

The estimation periods and the holding periods can be summarised as shown at the foot of Table 1. (The actual date of each estimation and holding period is shown in the table.) The Sharpe (1966) index is used to measure the performance during the holding period.

The required inputs to build the ex-post efficient frontier are the returns and the variance-covariance matrix of each of the sub-indices during the estimation period. Given the 500 observations of the returns on each index, ex-post mean returns for each index and the variance-covariance matrix can be obtained. These are then used as the inputs to find the ex-post efficient frontier. The various strategies that are tested and compared are shown in Figure 1.

The MVP is the portfolio on the efficient frontier that has the lowest risk. When the MVP strategy is used, the investor assumes that the only relevant information is the variance-covariance matrix of the asset returns. If the variance-covariance matrix is stable, the MVP is expected to have the lowest risk in the ex-ante period.

The optimal portfolio weights are derived as follows:

\[
R = (1 - w) Y + w Y_o
\]

where

\[R\] is the vector of predictive returns (N x 1 matrix; N = number of indices = 23);

\[Y\] is the N x 1 ex-post sample mean return vector;

\[Y_o\] is the mean return vector; and

\[w\] is the shrinkage factor.

The shrinkage factor is given by:

\[
w = \frac{(N+2) (T-1)}{(N+2) (T-1) + (Y-Y_o)' T S^{-1} (Y-Y_o)}
\]

Figure 1: Illustration of the various portfolio strategies

1. MVP
2. CETP
3. CETP/Rf
4. BSTP
5. BSTP/Rf

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The optimum portfolio weights are provided by the risk-free rate prevailing at the beginning of the holding period. In the extreme case where \( w = 0 \), the predictive return will be the return on the Minimum Variance Portfolio, and of the sample size. Given small sample sizes and large coefficients of variation of stock returns, portfolio analysis should not rely too much on the observed dispersion in sample means. This will be indicated by large values of \( w \). As the sample size increases as well as the estimated precision of the sample means, the shrinkage factor \( w \) decreases.

Figure 2 shows the efficient frontier with different values of the shrinkage factor \( w \).

With increasing shrinkage, the efficient frontier becomes flatter, and the expected increase in average returns dissipates accordingly. In the special case where \( w = 0 \), the predictive return is simply the ex-post sample mean return, which will be the same as the Certainty Equivalent Tangency Portfolio. In the extreme case where \( w = 1 \), the predictive return will be the return on the Minimum Variance Portfolio. In this case, diversification can only lead to risk reduction.

Having found the adjusted returns from Expression (1), the weights in the tangency portfolio based on BSTP can be obtained by specifying these adjusted returns and the unchanged variance-covariance matrix as the inputs — the difference being that the portfolio optimisation routine uses the returns which have been subjected to the adjustment in Expression (1).

The Bayesian Tangency Portfolio with the risk-free rate (BSTP/Rf), is similar to the CETP/Rf in that it uses the additional information provided by the risk-free rate prevailing at the beginning of the holding period. The optimum portfolio weights are obtained by the tangency point con-
connecting this risk-free rate to the shrunken efficient frontier.

As a further source of comparison we constructed an Equally Weighted Portfolio (naive). This naive portfolio is based on equal weighting of each index. This can be justified as an ad hoc method for controlling estimation risk.

A further benchmark return was provided by the Australian Accumulation All-Ordinaries Index. The performance of the portfolios was evaluated in terms of their returns and standard deviations in each of the 25 out-of-sample holding periods.

The risk-free rate used to calculate the optimal weights in the BSTP/Rf and the CETP/Rf, as well as the rate used in the Sharpe performance measure (with the risk-free rate), was the yield on 26-week Treasury notes. The annualised yield on the 26-week notes was converted to a daily rate, continuously compounded, using the following equation:

\[ r = \frac{1}{52} \ln \left[ 1 + \left( \frac{R_{26} \times 182}{182 \times 365} \right) \right] \]

where

\[ r = \text{daily yield for 26-week Treasury notes (continuously compounded);} \]

\[ R = \text{annualised yield for 26-week Treasury notes.} \]

In addition to the Sharpe measures of portfolio performance, the various portfolio strategies will also be ranked using dominance analysis.

RESULTS

The weights given to each asset by MVP are relatively unchanged from one period to another. (Details of the proportions invested in each of the holding periods for all five portfolio strategies will be made available by the authors on request). According to Jorion (1985) and Eun and Resnick (1988), this is due to the relative stability of the variance-covariance matrix of asset returns, and MVP relies only on the variance-covariance matrix of asset returns.

MVP is one extreme case when the value of the shrinkage factor \( w \) in Expression (1) is equal to one. On the other extreme, when \( w \) is equal to zero, estimation risk is totally ignored. The weights on this portfolio strategy are shown by the CETP. Due to the instability of the mean return vector, the weights are more unstable compared with the MVP.

The instability of the mean return vector shows the importance of the Bayes-Stein estimator.

The value of \( w \) found in deriving the BSTP mostly falls between 0.4 and 0.5 (it is not presented here but is available on request). The BSTP also depends on the mean return vector but it puts more emphasis on the variance-covariance matrix of asset returns.

The weights in CETP/Rf and BSTP/Rf, as suggested by Jorion (1985), are more unstable because of the introduction of a positive risk-free rate.

Weights given in one holding period may change significantly in the next holding period because of a slight change in the input estimates (especially the mean return vector).

Average Sharpe performance

Two Sharpe performance measures are reported. The first assumes a zero risk-free rate and will be referred to as SHP index (mean return of the portfolio divided by its standard deviation over the same period).

The second Sharpe measure uses a positive risk-free rate, is referred to as SHP/Rf index, and it is calculated as the mean excess return of the portfolio per one unit of the standard deviation.

Figures 3 and 4 show the average SHP index and average SHP/Rf index for each of the portfolio strategies over the 25 out of sample holding periods.

The MVP obviously performs better in terms of the Sharpe performance than any other strategies. In fact, it is the only strategy out of the five mean variance portfolio models that performs better than the equally weighted naive portfolio and the All-Ordinaries Index. The market during the sample holding period does not perform well, as the All-Ordinaries Index resulted in a negative SHP/Rf index. The MVP is the only portfolio strategy that has a positive excess return over the risk-free rate.

Conclusion

We examined the performance of various portfolio strategies in a domes-

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damental investors and noise traders can, depending on their relative weightings, lead to chaotic behaviour. The critical issue is to develop a better sense of the “onset of chaos” as this could offer real commercial advantages.

- Game theory, or “rational decision-making under uncertainty”. Like many mathematicians and economists, I have long wished that game theory would one day prove valuable. Two reasons for hope have emerged recently. Research on a game known as the “iterated prisoners’ dilemma” is yielding insights into behaviour that depends on the expected behaviour of competitors and draws an evident analogy with the behaviour of capital-market participants. Also, game-theoretic models of markets with imperfect and delayed information may improve modelling realism.

Behavioural aspects of investing are explicitly embedded in both of these. On the more positivist and technical side, windows of opportunity include new statistical techniques, with delightful acronyms like GARCH (generalised auto-regressive conditional heteroskedasticity), which offer improved techniques for modelling market volatility. Further, Evolutionary Neural Networks and artificial intelligence in general, hype notwithstanding, offer potentially powerful pattern-recognition techniques. The Economist article discusses this in some (non-technical) detail.

Whatever the future, supply and demand for quants will continue to be governed by a version of Stiglitz’ paradox: quants are necessary to discover market inefficiencies which, once discovered, are quickly arbitrated away — making quants unnecessary!

And whatever the future, quants will need to pay heed to G.K. Chesterton’s apposite warning: “The real trouble with this world of ours is not that it is unreasonable nor even that it is a reasonable one. It looks a little more mathematical and regular than it is; its exactitude is hidden . . . The commonest kind of trouble is that it is nearly reasonable but not quite . . . Its wildness lies in wait.

NOTES
1. It is sobering to recall that in thirteenth-century Pisa the use of this numeration system was forbidden on pain of death. In its wisdom, the Church continued using the Roman numeration system for book-keeping until the nineteenth century. The lesson regarding resistance to new ideas should not be lost on the finance industry.

2. Synthetic instruments have an ancient lineage. “Absolution futures” were sold by a twelfth-century Pope. Options flowered briefly during the sixteenth-century tulip bulb craze in the Netherlands.

REFERENCES