Court actions delayed the introduction of the Australian Stock Exchange's new-age instrument, the low exercise price option, while arguments continued about its true identity. Is the LEPO an option or a futures contract? Christine Martini proposes that theoretical pricing models may help to suggest an answer.

On 16 May 1994 the Sydney Futures Exchange launched its share futures contract. Long before the SFE launch, the Australian Stock Exchange had been planning new derivative products. One was to be a "low exercise price option", or LEPO. Some discussion developed in the financial markets about whether a LEPO was in fact an individual share-price futures contract. Despite a legal challenge by the SFE, on 2 November 1994 the Federal Court ruled that LEPOs could trade on the Australian Stock Exchange.

This article explores the theoretical pricing models appropriate for LEPOs and their relationship to the accepted models for valuing options and futures. LEPOs can in fact be priced using an option valuation framework or a futures valuation model, and the two approaches can be shown to be equivalent under certain assumptions. The use of LEPOs as a hedging tool is discussed, with particular reference to the hedge parameters derived from option pricing models.

The main characteristics differentiating LEPOs from exchange-traded stock options are:

- LEPOs are European options, implying that they can only be exercised at maturity. Most exchange-traded options are American-style and can be exercised at any time up to and including the maturity date.
- LEPOs have a very low exercise price — a typical exercise price might be 10 cents. Thus these options will be in-the-money options and have an almost 100 per cent probability of expiring in-the-money.
- For each stock on which LEPOs are listed, there will be only one exercise price, typically with maturity dates set each month.
- Perhaps the most important distinction is that the premium on other stock options traded on the Australian Options Market is payable up front. In contrast, the premium for LEPOs will be margined and the margin account will be marked to market throughout the life of the option. This means that the buyer of a LEPO will post a risk margin, and then the risk margin and the mark-to-market account will be adjusted daily.

Pricing models

The no-dividend case

The second and last features of the contract specifications for LEPOs imply that the derivative instrument will behave more like a futures contract than an option contract. In deriving an appropriate option pricing valuation equation, let us assume that the premium is paid up front in order to explore how the Black-Scholes Option Pricing Model prices such a low exercise price option. The standard Black-Scholes prices a European call option on a non-dividend paying stock as

\[ C = S \frac{d_1}{N(d_1)} - Ke^{-rT}N(d_2) \]  

(1)

where

\[ d_1 = \frac{\ln \left( \frac{S}{K} \right) + (r + \frac{1}{2} \sigma^2 T)}{\sigma \sqrt{T}} \]

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\[ d_2 = d_1 - \sigma \sqrt{T} \]

The call premium \( C \) is related to the stock price \( S \), the exercise price \( K \), the risk-free interest rate \( r \), time to maturity \( T \) and the volatility of the stock. \( N(.) \) denotes the area under the normal probability density function and \( N(d_1) \) and \( N(d_2) \) can be viewed as providing some measure of the probability that the option will finish in the money. Intuitively, the holder of a call option will receive the stock if and only if the call finishes in the money, whereas, in order to obtain the stock the holder of the call option must pay the strike price, \( K \), if and only if the call finishes in the money.

Looking at the equation for \( d_1 \), we observe that if \( K \) is very small relative to \( S \), then \( S/K \) is very large, and the logarithm of \( S/K \) will also be quite large. Therefore the probability measures \( N(d_1) \) and \( N(d_2) \) will be very close to 1. This implies that for very low exercise price options the Black-Scholes equation reduces to a simple form:

\[ C = S \cdot Ke^{-rT} \quad (2) \]

The call option price no longer depends on the volatility of the stock because the probability of the option finishing in the money is close to 1.

Suppose that instead of the payment of an up-front premium for the option, a system of margin deposits and calls exists, not unlike that of a futures contract. The price of this margin low exercise price option will be higher, because the buyer is no longer tying up funds and the seller no longer receives the premium. The taker of the LEPO contract receives the benefit of the option premium being deferred in time. In fact, when the funds in the margin account earn the risk-free rate, the margin account earns the risk-free rate over the time. In fact, when the funds in the margin account earn the risk-free rate, the margin account earns the risk-free rate over the time. In fact, when the funds in the margin account earn the risk-free rate, the margin account earns the risk-free rate over the time. In fact, when the funds in the margin account earn the risk-free rate, the margin account earns the risk-free rate over the time.

\[ L = S \cdot e^{rT} \quad (3) \]

This is equation (3) again, derived this time using a futures valuation approach. That is, the two approaches give exactly the same valuation equation. It is essentially showing that the LEPO is valued similarly to a futures contract and therefore will behave more like a futures contract than an option contract. For example, its price will be insensitive to the volatility of the underlying share price. The volatility is an important determinant of all option prices, but will have virtually no impact on the price of a LEPO.

The dividend case

If the underlying stock pays a single dividend \( D_t \) at time \( t \) then the Black Scholes model is adjusted to

\[ C = (S - D_t^*) \cdot N(d_1) - Ke^{-rT}N(d_2) \quad (4) \]

where

\[ d_1 = \ln \left( \frac{S - D_t^*}{K} \right) + \left( r + \frac{1}{2} \sigma^2 \right) T \]

\[ d_2 = d_1 - \sigma \sqrt{T} \]

and \( D_t^* \) is given by the discounted value of the future dividend, that is

\[ D_t^* = D_t e^{-rT} \]

Following the same arguments as for the no-dividend case, the fair price for the LEPO will be

\[ L = (S - D_t^*) e^{-rT} - K \quad (5) \]

The case of multiple dividends will be a simple extension of this case. Once again the valuation equation (5) could be derived using either the options valuation approach or a futures valuation approach.

Hedge parameters

When options are used to hedge exposure to the underlying asset, then a number of sensitivity parameters become important in managing the risk of the position. The delta of the option is perhaps the most important hedge parameter, where delta measures the sensitivity of the option price to movements in the underlying asset when all the other variables that the option price depends upon are held constant. Other important sensitivity parameters are gamma and vega.

\[ \Delta = e^{-rT} \]

For example, if the risk-free rate is 6 per cent and the time to maturity of the option is 0.5 years, then

\[ \Delta = 1.03 \]

This implies that for every dollar change in the price of the share the price of the LEPO will increase by $1.03. Of course, since each LEPO contract covers 1,000 shares there will be an increase in the value of each LEPO contract of $1030 for each $1 increase in the share price. This illustrates the high leverage afforded by the contract.

\[ \Gamma \]

The gamma of an option position measures the change in the delta as the underlying asset price changes. Effectively it gives a measure of how often the hedge ratio will have to be changed in order to remain hedged.
Because the delta does not depend on the stock price, the gamma of the LEPO is zero. Using LEPOs to hedge will not involve continual adjustment to the hedge ratio; hedging using these instruments will be much more like a "hedge and forget" scheme.

Vega
The vega measures the sensitivity of the option price to changes in the volatility of the underlying share price. Because the LEPO price is independent of volatility, as shown in equation (3), the vega of the LEPO contract will be zero.

The LEPO therefore behaves more like a futures contract than an option contract. Because the strike price in the LEPO contract is set so low, typically 10 cents or less, over the most likely range of share prices the price of the LEPO is a linear function of the share price.

The price of the share futures contract on a non-dividend paying stock would simply be given by

$$ F = S_0 e^{rT} \quad (6) $$

A comparison of this equation with equation (3) shows that the difference between a LEPO and a share future is simply the strike price paid on the LEPO, which will be typically very low (of the order of $10 to $100 depending on where the exercise price is set).

Example
On 5 August 1994 the shares of ABC Ltd are trading at $19.60, with volatility estimated at 21 per cent p.a. and a risk-free rate in the market of 5.00 per cent p.a. December LEPOs with a strike price of $0.10 are also trading. Time to maturity is 146/365 = 0.4 years.

Using equation (3) to value the LEPO gives a LEPO premium of

$$ \text{LEPO} \, \text{value} = \frac{1}{2} \times \text{volatility} \times \text{strike} \times \text{delta} \times \text{time to maturity} = 0.21 \times 0.10 \times 0.4 = 0.084 $$

**The LEPO therefore behaves more like a futures contract than an option contract.**

$19.90. Assume that the LEPOs are trading in the market place at their calculated fair price of $19.90 and that on 29 December the share price has risen to $21.50. To compare investment in the shares and investment in the LEPO contract, suppose that investor A purchased 1,000 shares of ABC Ltd on 5 August and sold them on 29 December, while investor B purchased 1 ABC LEPO, held the option until expiry and exercised it. To make the two situations directly comparable, assume that investor B immediately sells the shares. The cashflow implications of the two positions are given in Table 1 (transaction costs are ignored).

While it is not possible to compare accurately the return on the two investments without knowing all the intervening cashflows for the LEPO, this simple example gives an idea of the leverage involved in investment in the LEPO contract. Ignoring intermediate cashflows, the investment in the LEPO has given a return of 150 per cent over the period that the option was held, while investment in the shares yielded 9.6 per cent over the same holding period.

Conclusion
With similar margining sytems in place and contracts covering the same number of shares, a LEPO contract and a share futures contract can be shown to be similar financial instruments. The fact that the ASX is looking to its new technology to provide new risk management products for equity investors, coupled with the SFE's desire to move more into equity derivative products, has led to almost identical products being developed in the two markets. The ASX may have had some advantage with the introduction of LEPOs because the margin requirements can be netted and met with shares certificate deposits, whereas the SFE requires cash deposits for margins and margin calls.