Mathematics has always had a role in finance and investment, and as the mathematics has changed, so has the role.

Jack Gray examines the evolution of mathematical theories and applications, their relevance to investment strategies and the inescapable truth that, no matter how much certainty mathematics can seem to bring to financial markets, everything turns on human behaviour.

Since the first recorded use of compound interest by the rivers of Babylon some 4,000 years ago, finance and investment have been mathematically intensive, albeit at elementary levels. By contrast, the past quarter-century has been distinguished by a veritable explosion in the use of sophisticated mathematics. Not only is the mathematics itself sophisticated; the manner of its use and the issues it raises also exhibit demanding levels of sophistication.

Those of us trying to apply mathematics — or quantitative techniques — in the world of investments are called “quants”, presumably because the word mathematics fills the heart with terror. Some US fund managers proudly trumpet their mathematical styles as “Geometry Asset Management” or “Martingale Asset Management”. I eagerly await the entry of the “Chaotic Asset Management Company”.

The quant style is typified by index funds that rely on mathematical techniques such as sampling and optimisation. By “replicating” a market index, the All Ords for instance, these funds guarantee performance of that index. They are labelled “passive” because ideally they require neither human decision-making nor active trading. Consequently they can operate at relatively low cost.

About 20 per cent of US pension funds manage their equity sectors in passive index funds. Although this may increase in the current environment of moderate growth and cost-sensitivity, most funds will probably continue to be actively managed. As active equity managers in the US tend to underperform the S&P 500, economic rationalists are confronted with a problem: why do US pension funds continue to choose active management, which produces lower returns and greater costs than passive management? Both quant and non-quant answers are available.

On the quant side, there is evidence that active management does produce lower risks of short-term volatility and negative returns than passive management. Active managers can avoid negative returns by anticipating poorly performing sectors and by using derivatives. On the non-quant side, investors appear to be willing to pay a premium for regular market intelligence and forecasts passed on by their fund managers — information generated by active managers but not by passive managers.

These investors would not be “rational risk-adjusted return maximisers”, as is assumed in most models of the capital markets. Their preference highlights a fundamental problem with mathematical modelling: investment decisions depend critically on human behaviour.
Mathematics in finance and investing

A number of disparate factors account for the growing use of sophisticated mathematics in finance and investments. These include:

- The rise of institutional investors who see the potential for added value, who can afford quants and who are under severe competitive pressures. Marginal enhancements to risk-adjusted returns, capable of shifting a manager from the second quartile to the first in performance comparisons, can have a dramatic impact on business.
- The growing complexity of the investment industry, driven by changing demographics and the need for retirement income policies; simultaneous deregulation of financial markets and increasing regulation of retirement markets; internationalisation; and increasingly sophisticated clients.
- The availability of fast, powerful, cheap desktop computers which, through simulation, enable the use of more realistic models of capital markets. However, their power comes at the cost of a loss of certainty. Beyond the question of program correctness lies the more serious question of program validation: how can one ensure that a program is an accurate representation of a model? (Continual testing is a less-than-adequate solution. I am far more comfortable when garbage comes out, and I know the program is wrong, than when "reasonable" answers appear and I am in doubt!) This problem will be exacerbated by the next generation of computers, which will introduce a degree of artificial intelligence.
- The drift of mathematicians into non-traditional and more employable areas. This phenomenon may receive a boost from the peace dividend as mathematicians in the US and the ex-USSR are released from defence-related work. Also, in the new "united" Europe, pension funds are being allowed the luxury of significant exposure to equities. Particularly in Germany, with its strong mathematical and commercial traditions, this could trigger a powerful marriage of mathematics and finance.

The value added by mathematics

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The development of algebra (including zero and negative numbers), about AD 1500, and the related use of graphs and charts. This was an enormous leap in communicating and understanding quantified data. How best to turn data into meaningful information and communicate it retains significant commercial currency and is a further way in which quants can add value. After nearly half a millennium of research and development into algebra, a technique known as linear programming — a peace dividend of the Second World War — led directly in the 1950s to the cornerstone of modern investing, Markowitz' Modern Portfolio Theory. Through quantification, Markowitz forced serious attention on to risks. So dominant is this view that today fund managers present and organise themselves as managers of risk as much as of return.

The development of probabilistic ways of thought, ie, the formalisation of uncertainty, in the seventeenth century. Modern probability theory provided the tools Fischer Black and Myron Scholes needed to develop their formula for valuing options in the early 1970s.

Descriptive and interpretative statistics, ie, the analysis of variability, dating from about 1880.

Mathematics used critically

An excellent example of how higher mathematics adds value comes from Modern Portfolio Theory. Mathematical analysis transforms a slogan for diversifying ("Don't put all your eggs in one basket") into a hard set of decision criteria for the why, how and extent of diversification.
But mathematics can also subtract value. Its intrinsic beauty, supposed ineluctability and precision make it highly seductive and hence potentially dangerous. I constantly remind myself of a wise dictum of John Maynard Keynes: "It's better to be vaguely right than precisely wrong."

Real value resides in the critical discipline mathematics can instil. Because markets are driven by human emotions (fear and greed, but never guilt), they exhibit extreme levels of uncertainty. The consequence is a plethora of reasonable but opposing views that can act against discipline.

By forcing attention on to underlying assumptions, mathematical models offer a rational way of approaching decisions and responding to the lure of commercial opportunities. This is no better illustrated than by Fischer Black's critical re-examination of his option pricing model some fifteen years after its creation, and by the serious reservations that Fama and French, fathers of the efficient market hypothesis, have expressed about their offspring's validity twenty years after its birth.

Paradoxically, the power of a mathematical model may derive from attempts to falsify its conclusions. This can occur on two fronts. First, the process of arguing why a conclusion may be false leads to deeper and more critical levels of understanding. Second, knowing how a conclusion may fail exposes the risks involved in making decisions based on that conclusion.

The importance of an extremely critical approach to the use of mathematics in the financial arena holds true in the physical sciences too, but less so. There, as the Nobel prizewinning physicist Eugene Wigner observed, mathematics is "unreasonably effective". It is as if, as Galileo put it, "the book of nature is written in the language of mathematics".

I do not believe the books of finance, economics or investment are written in that language, because the Newtonian paradigm of observing, modelling the driving mechanisms, experimenting, testing and predicting is largely inappropriate in these fields. There may not be any fundamental mechanisms driving markets; meaningful experiments cannot be conducted; and, even more problematic, predictions, the very output of models, can be invalidated quickly by human actions. The "no-arbitrage" argument is a powerful critical component of mathematical finance.

Newton seemed to recognise these difficulties when he observed with frustration that he could "calculate the motion of heavenly bodies, but not the madness of people". He made his complaint in 1720 at the time of the South Sea Bubble, during which he lost the equivalent of a million dollars. Can mathematical models ever be robust enough to account for the madness of crowds? There have been some (humble) attempts to do so, to apply sophisticated mathematics to behavioural aspects of the economic and social sciences. In the mid-1970s, catastrophe theory was in vogue and found supposed applications to the stockmarket and to prison riots.

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The positivist approach also dominates performance measurement of investment managers' returns. The noisy, dirty nature of the data and small relative outperformances mean that 50 or more years of data are needed to ensure statistical significance.

Nonetheless, investors continue to rely almost exclusively on past performance as a predictor of future performance. The appeal of this is that the information is "hard" and quantified compared with other "soft" and qualitative, yet possibly more reliable, predictors. This is a manifestation of the "bean counter's view of the world": everything should be measured — even those things that can't be. Surprisingly, this is the antithesis of the quant's worldview. Quants often add value by warning of the dangers of excessive and inappropriate quantification.

The future

As markets and regulations become more complex and as the population density of quants (and their influence in institutional investment organisations) increases, quant activity will no doubt expand. These developments already look promising:

- Chaos theory, or non-linear dynamical systems. The 9 October issue of The Economist reports on commercially successful chaos-based foreign-exchange trading systems being run with real money. At a more theoretical level, intuitively appealing market models consisting of a mix of fun-
damental investors and noise traders can, depending on their relative weightings, lead to chaotic behaviour. The critical issue is to develop a better sense of the "onset of chaos" as this could offer real commercial advantages.

**Game theory, or "rational decision-making under uncertainty".** Like many mathematicians and economists, I have long wished that game theory would one day prove valuable. Two reasons for hope have emerged recently. Research on a game known as the "iterated prisoners' dilemma" is yielding insights into behaviour that depends on the expected behaviour of competitors and draws an evident analogy with the behaviour of capital-market participants. Also, game-theoretic models of markets with imperfect and delayed information may improve modelling realism.

Behavioural aspects of investing are explicitly embedded in both of these. On the more positivist and technical side, windows of opportunity include new statistical techniques, with delightful acronyms like GARCH (generalised auto-regressive conditional heteroskedasticity), which offer improved techniques for modelling market volatility. Further, Evolutionary Neural Networks and artificial intelligence in general, hype notwithstanding, offer potentially powerful pattern-recognition techniques. The *Economist* article discusses this in some (non-technical) detail.

Whatever the future, supply and demand for quants will continue to be governed by a version of Stiglitz's paradox: quants are necessary to discover market inefficiencies which, once discovered, are quickly arbitrated away — making quants unnecessary!

And whatever the future, quants will need to pay heed to G.K. Chesterton's apposite warning: "The real trouble with this world of ours is not that it is unreasonable nor even that it is a reasonable one. It looks a little more mathematical and regular than it is; its exactitude is hidden... The commonest kind of trouble is that it is nearly reasonable but not quite... Its wildness lies in wait."

**REFERENCES**


