Consultants’ attribution analyses of fund performance can be in some cases misleading. In most cases, they could be improved by tailoring the measurement strategy and methodology to suit the fund under consideration. In this Quant Report, Les Balzer and David Starer consider two aspects of attribution – first, the importance of the interaction effect and, second, how to attribute to multiple sources of return and not just to market timing and security selection.

If a fund holds an index or benchmark portfolio in a particular asset class but overweighted that sector, the additional return is clearly attributable to the asset allocation (overweighting) decision. If, on the other hand, a fund neither overweighted nor underweighted a sector, but chooses to hold a non-benchmark portfolio, the effect on the return is clearly a security selection effect.

In normal circumstances, however, both asset allocation and security selection will differ from their benchmarks at the same time. In this case, not only do the above individual effects occur but a combined or interaction effect also arises. This occurs because any overweight portion of the portfolio consists of a non-benchmark selection of securities and earns at a non-benchmark rate. Similarly, any underweighting detracts from portfolio earnings at a non-benchmark rate.

The interaction effect occurs because two things have happened simultaneously and not separately, and because their effects are not additive. Mathematically speaking, the situation is non-linear (multiplicative in this case). This non-linearity means that the interaction effect cannot be broken into two separate components and added to the asset allocation and security selection effects. There is no simple mathematical basis for adding the interaction effect to either or both basic effects using a single-period model.

In Figure 1, the area of the large rectangle (bottom left) represents the total return which would have been achieved with a benchmark asset allocation and a benchmark (usually index) security selection. The top left rectangle represents the incremental return arising from deviating away from the benchmark asset allocation, while the bottom right rectangle represents the value added by moving away from the benchmark security selection. The rectangle in the top right represents the interaction effect.

The interaction rectangle could be arbitrarily assigned to asset allocation or to security selection, or distributed between the two in some systematic way. It must be realised, however, that doing so is a completely arbitrary act and has no scientific or mathematical basis in a single-period model. It can also be quite misleading.

Example 1

Consider this example. A fund manager thinks that the equities market is overheated and due for a significant correction. The manager not only selects defensive stocks but also hedges half of the equity exposure with over
the-counter forwards, which match the portfolio exactly. In the event, the manager is wrong and the market rises 10 per cent (as measured by the benchmark index) and the unhedged portion of the portfolio rises 15 per cent due to superior stock selection.

If \( W_a \) and \( W_b \) represent the actual and benchmark weightings, respectively, and if \( R_a \) and \( R_b \) represent the actual and benchmark security returns, respectively, then \( W_a = 0.5 \), \( W_b = 1 \), \( R_a = 15 \) per cent and \( R_b = 10 \). Any cash effects are ignored.

Hence

\[
W_a \text{ portfolio return} = W_a R_a = 0.5 \times 15\% = 7.5\% \tag{1}
\]

Benchmark portfolio return = \( W_b R_b = 1 \times 10\% = 10\% \tag{2} \)

and the return from active management is given by

\[
\text{Active return} = W_a R_a - W_b R_b = 7.5\% - 10\% = -2.5\% \tag{3}
\]

The purpose of attribution analysis is then to determine the source(s) of the active return.

To perform the attribution analysis, we need to note that the actual weighting can be thought of as the benchmark weighting plus a deviation, \( \Delta W \), away from the benchmark, so that

\[
W_a = W_b + \Delta W \tag{4}
\]

Similarly, the actual return can be thought of as the benchmark return plus a deviation, \( \Delta R \), away from the benchmark return, so that

\[
R_a = R_b + \Delta R \tag{5}
\]

Consequently, the actual portfolio return, \( R_p \), is

\[
R_p = W_a R_a = (W_b + \Delta W)(R_b + \Delta R) = W_b R_b + \Delta W R_b + W_b \Delta R + \Delta W \Delta R \tag{6}
\]

A little thought should reveal that: Benchmark return = \( W_b R_b \) \tag{7}
Asset allocation effect = \( \Delta W R_b \) \tag{8}
Stock selection effect = \( W_b \Delta R \) \tag{9}
Interaction effect = \( \Delta W \Delta R \) \tag{10}

In this case,

\[
\Delta W = W_a - W_b = 0.5 - 1.0 = -0.5, \quad \text{and} \quad \Delta R = R_a - R_b = 15\% - 10\% = +5\%
\]

and hence

\[
\text{Asset allocation effect} = \Delta W R_b = -0.5 \times 10\% = -5\% \\
\text{Stock selection effect} = W_b \Delta R = 1.0 \times 5\% = +5\%
\]

Interacting effect = \( \Delta W \Delta R = -0.5 \times 5\% = -2.5\%

So, the asset allocation decision alone would have cost five percentage points of return, while the stock selection decision alone would have added five. Separately, they would have cancelled each other out. But because they were implemented simultaneously, their interaction caused a further loss of two and a half percentage points. Their combined effect equals the total active return.

Arbitrarily adding the interaction effect to the asset allocation effect leads to the latter appearing to be an even more painful -7.5 per cent; while arbitrarily adding it to the stock selection effect makes that effect appear to be less valuable at only +2.5 per cent instead of +5 per cent. Unfortunately, many analysts do one or the other, or even a combination of both, without informing the reader. Such a situation is totally unacceptable. The interaction term must be shown separately in any attribution analysis.

**Attributing to multiple sources of return**

Some analysts are currently attributing to "market timing" and to "stock selection". Stock selection is obviously the equities sector equivalent of the more generalised notion of security selection. Market timing contributions are assessed by performing an attribution analysis relative to the fund’s own benchmark asset allocation. This is clearly useful. However, it fails to expose the potentially important contribution made by the fund’s benchmark asset allocation relative to those of similar funds. Such an analysis would involve attributing to more than one asset-allocation related decision. How would such an analysis be performed?

We begin by noting that the actual asset allocation can be thought of as consisting of three components instead of two.

The first might be an allocation which is representative of the "universe" of similar funds. A robust measure of the typical or "central" value is the median. The median is simply the number which falls in the middle when all the figures are sorted by size. It is not distorted by occasional large or very small numbers. We shall denote the median asset allocation for the universe of similar funds by \( W_u \). Next, the difference between the fund’s own benchmark or strategic asset allocation and the median is denoted by \( \Delta W_{saa} \), so that

\[
W_b = W_u + \Delta W_{saa} \tag{11}
\]

The difference between the fund’s actual asset allocation and its benchmark or strategic asset allocation can then be thought of as a market timing decision or deviation \( \Delta W_{mt} \), so that

\[
W_a = W_b + \Delta W_{mt} \tag{12}
\]

Using equations (11) and (12), the actual return can then be decomposed into three components

\[
W_a = W_u + \Delta W_{saa} + \Delta W_{mt} \tag{13}
\]

The portfolio return is then

\[
R_p = W_a R_a = (W_u + \Delta W_{saa} + \Delta W_{mt})(R_b + \Delta R)
\]

\[
= W_u R_b + \Delta W_{saa} R_b + \Delta W_{mt} R_b + W_u \Delta R + \Delta W_{saa} \Delta R + \Delta W_{mt} \Delta R
\]

\[
= (W_u + \Delta W_{saa}) R_b + \Delta W_{mt} R_b + (W_u + \Delta W_{saa}) \Delta R + \Delta W_{mt} \Delta R
\]

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A fund which adopts the median asset allocation of similar funds and which selects securities along index lines can hardly be said to be adding value. Hence it seems reasonable to define "value added" as the difference between the actual return and that available from the median asset allocation and an index portfolio. Thus if the sector benchmark is the sector index, then

\[
W_a = W_a R_a + \Delta W_{sat} R_b + \Delta W_{mt} R_b + \Delta R + \Delta W_{mt} \Delta R \quad (14)
\]

Close inspection of equation (15) reveals that the returns attributable to sources of interest are:

- Strategic asset allocation effect = \(\Delta W_{sat} R_b\) \(\Delta R + \Delta W_{mt} \Delta R\) (16)
- Market timing effect = \(\Delta W_{mt} R_b\) (17)
- Stock selection effect = \(W_b \Delta R\) (18)
- Interaction effect = \(\Delta W_{mt} \Delta R\) (19)

Example 2

Consider a simplified situation where a fund invests in only equities and fixed-interest securities. The median asset allocation for the universe of similar funds is equities 68 per cent: fixed interest 32 per cent. The fund of interest has a less optimistic medium-term view of the world and local economies and sets a strategic benchmark allocation of equities 62 per cent: fixed interest 38 per cent. In the short term, however, a tactical decision is taken to increase equities to 64 per cent and to reduce fixed interest to 36 per cent. At the end of the period, the fund has returned 14.16 per cent pa, whereas a median asset allocation and index returns would have produced 12.88 per cent pa. Where has the 1.28 per cent additional return come from?

The fund’s own sector benchmarks were in fact simply the sector indices, which returned 10 per cent pa and 19 per cent pa for equities and fixed interest. Although these returns are typical of the early nineties, that fact is irrelevant to explaining the power of the multiple-source attribution technique. The fund’s equity and fixed-interest portfolios returned 12 per cent pa and 19 per cent pa respectively.

Using equations (15) to (19) for each sector individually and adding the results, the multiple-source attribution reveals the sources of gains and losses as shown in Table 1.

Overall, the fund’s selection of a medium-term strategic asset allocation different from its peer group cost it 0.60 per cent in equity return but added 1.14 per cent to fixed interest returns, leading to a worthwhile net gain of 0.54 per cent.

Its short-term tactical diversion from its strategic asset allocation produced a net loss of 18 basis points for the fund.

Security selection was the major overall source of value added at 0.86 per cent. This disguised the fact that security selection was quite good for equities (+1.24 per cent) but poor for fixed interest (-0.38 per cent).

Both the equities and fixed-interest portfolios contributed soundly to the 1.28 per cent value added by the fund. Equities did this predominantly through security selection, while strategic overweighting of fixed interest was the major source of its contribution.

Since the interaction effects are individually and collectively small, all of the above comments can be regarded as reliable.

Further generalisation

To this point, only asset allocation has been generalised. It is fairly obvious, however, that return can also be generalised. For example, a fund might use a sector benchmark which differs from a more widely used market index.

An analysis which splits the actual return into an index return, a component arising from the difference between the index and the benchmark, and finally a security selection component might be of interest.

In general, both asset allocation and sector return can be split into a base figure and any number of meaningful variations away from it. For example, using a three-way decomposition for both weighting and return:

\[
\text{Value Added} = W_a R_a - W_b R_b
\]

A suitable grouping and interpretation of the terms in this equation will depend on how the base weightings and returns, and the deviations from them are defined. One of many possible interpretations is:

\[
\text{Value Added} = W_a R_a - W_b \text{R}_{\text{base}}
\]

Conclusions

The interaction term, which unavoidably arises in attribution analysis, should not be added to any of the other effects. Doing so will produce distortions. Further, the relative size of the interaction effect gives an indication of the reliability the other effects exposed by the attribution analysis.

Attributes to “market timing” and “security selection” are only part of the story. An obviously important additional component is the value added (or subtracted) by the strategic asset allocation decision.

But these are not the only effects of interest in all cases. Hence, just as the bespoke tailor fits a suit to the individual’s needs, so too can attribution analysis be fitted to suit the needs of individual situations.