The importance of estimating prepayment risk when valuing mortgage-backed securities is highlighted by market experience over recent years. ANDREW ANG discusses a model which predicts the likelihood of prepayment and the premium required by investors to compensate for this risk.

Mortgage-backed securities (MBSs) are a term-structure derivative product, paying out regular coupons backed by a pool of insured mortgages. In Australia the MBS market is still small, compared with its counterpart in the US, and approximately half of all outstanding issues consist of FANMAC bonds, issued by the NSW state government. The FANMAC market is of low liquidity and is restricted to institutional investors.

There are three types of MBS: pass-through, pay-through and collateralised mortgage obligations (CMOs). With pass-through MBSs, investors are exposed to the risk of early repayment of the mortgages. The prepayments are passed-through when the coupon payments are made, the coupon payments themselves being dependent on the remaining balance in the underlying pool. Investors holding pay-through MBSs are insured from this prepayment risk; these act like conventional bonds. CMOs are derivative instruments of MBSs which package the cashflows from the mortgage pool in different ways.

FANMAC bonds are pass-through securities, so investors are exposed to prepayment risk. The underlying mortgage pools consist of two types of mortgage: “Low-start” and “Affordable” home loans, both issued to low-income earners. The mortgages are initially negatively amortising (loan instalments do not cover the interest, so principal increases), with increasing instalments and a fixed rate.

Twenty-three trusts have been issued, each containing different proportions of Low-starts and Affordables, and within each trust the mortgages themselves have different principal amounts and payments. The mortgages are supported by the NSW government, and any defaults are passed through to the investors in the same way as prepayments.

FANMAC suffered severe setbacks in the early 1990s because of heavy prepayments of its bonds. These arose from rapidly falling interest rates which encouraged borrowers to pay off their mortgages. These prepayments were passed through to investors, who suffered large capital losses. The episode has illustrated the importance of properly estimating prepayment risk when valuing FANMACs and any other mortgage-backed security.

WHAT AFFECTS PREPAYMENTS?
The basic elements of a prepayment process can be divided into rational financial decisions, such as favourable refinancing opportunities, and non-financial circumstances such as death, divorce, loss of income, changing housing requirements or job relocations. These are described as systematic factors (due to interest rates) and unsystematic factors.

Prepayments affect the duration and convexity of MBSs. For a normal non-prepayable bond, duration (the weighted average maturity of a security) decreases as interest rates rise because later payments receive less weight in a present-value sense than the earlier cashflows. With MBSs, prepayments cause duration to increase as rates rise because less weight is placed on early cashflows when rates are high. Thus movements in interest rates cause a twofold change in duration:
1. Change due to interest rates:

\[ i \uparrow \Rightarrow D \downarrow \text{ and } i \downarrow \Rightarrow D \uparrow, \]

which occurs for a normal bond, and

2. Change due to prepayments:

\[ i \uparrow \Rightarrow \text{prepayments} \downarrow \Rightarrow D \uparrow \text{ and vice versa.} \]

These effects act in opposite directions. The second effect may be stronger than the first, causing negative convexity (change in duration). MBSs may have negative convexity because the prepayment effects swamp the interest-rate effects on duration.

THE PSA MODEL

The current standard for measuring FANMAC prepayments is the PSA (Public Securities Association) model imported from the US. The PSA uses the conditional prepayment rate (CPR) measure. This is an annualised rate of prepayments conditional on the previous month’s remaining balance. This accounts for the compounding effect of monthly prepayments. The corresponding monthly rate is the Single Monthly Mortality Rate (SMM). These are related by:

(1) \[ \text{SMM} = 1 - (1 - \text{CPR})^{1/12} \]

If \( B_t \) is the actual balance of the mortgage at the end of the month \( t \), \( B_t^* \) is the scheduled balance at the end of month \( t \) conditional on \( B_{t-1} \), then \( B_t^* - B_t \) measures the dollar amount of the prepayments in month \( t \). If there are no prepayments, \( B_t^* = B_t \). The SMM in month \( t \) is:

(2) \[ \text{SMM}_t = \frac{B_t^* - B_t}{B_t} \]

and we find \( B_t^* = (1+c)B_{t-1} - P_t \), where \( P_t \) is the actual mortgage payment for month \( t \), and \( c \) is the monthly interest rate on the mortgage. We can illustrate this as in Figure 1.

We can then calculate the survival factor, \( SF_t \), of the scheduled balance that is outstanding in any given month by:

(3) \[ SF_t = 1 \quad \text{for } t = 0 \]

(4) \[ SF_t = SF_{t-1} \times \text{SMS}_t \quad \text{for } t > 0 \]

where \( \text{SMS}_t \) is Single Monthly Survival = \[ \text{SMM}_t \].

The PSA model is expressed as a monthly series of CPR rates. It begins at 0.2 per cent per year in the first month and increases by 0.2 per cent per year in each successive month until month 30, where it levels out at 6 per cent. Hence the PSA standard is a series of CPR rates. Prepayment experience is quoted in multiples of the FHA or PSA standards, eg 100 per cent PSA is the standard PSA model, 200 per cent PSA for the second month corresponds to a CPR of 0.8 per cent per year.

FANMAC prepayments are particularly hard to model because within each trust each mortgage type has mortgages grouped in one or more pools with different origination dates. This means that each pool is exposed to a different series of interest rates and is of a different maturity, so each pool has a different prepayment pattern. Each pool has a different survival factor within the same trust and the same type of mortgage.

To overcome this, a unique survival factor table for each pool was constructed using a conditional prepayment model from the inception of each pool. At an observation date the remaining portion of each pool, given each pool’s unique survival factor, was aggregated, and this was compared with the actual balance of the mortgage type outstanding.

Non-systematic factors were captured using a base-line function which modelled prepayments not dependent on the refinancing abilities of mortgagors. For most MBS issues, especially FANMACs, the amount of prepayments dependent on interest-rate factors is very important and is not captured by a standard PSA model.

A FANMAC PREPAYMENT MODEL

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Figure 1
level of the current interest rate. Interest-sensitive prepayments were modelled using a factor which increased the base-line function if interest rates were higher than the mortgage rate.

Following Green and Shoven (1986), we use a base-line function $A(t)$, and the interest-sensitive loading $n(t)$:

(5) $\text{SMM}_t = A(t) n(t)$

We employ a log-logistic hazard function as the base-line function for SMM:

(6) $\gamma(t) = \frac{\gamma p(t)}{1 + (\gamma p)^{n}}$

This was chosen because this function can take on a variety of shapes by varying its two parameters $\gamma$ and $p$. The function increases from 0 at $t = 1$ to a maximum at $t^* = (p-1)^{1/p}/\gamma$, and this reflects the behaviour of interest-rate invariant prepayments being low in the first years of a mortgage, increasing and then decreasing as burnout takes effect. As Schwartz and Torous (1989, p 379) note, this type of function enables us to incorporate a priori knowledge of prepayment behaviour. Interest-sensitive prepayments are captured using the function:

(7) $n(t) = \exp \left( (\beta_1 (\text{mortgage rate} - r) + \beta_2 \right)$

for $r < \text{mortgage rate}$, otherwise $= 1$

where the mortgage rate is the average weighted rate of the underlying mortgages (which FANMAC provides) and $r$ is the current interest rate. As time passes interest sensitivity decreases because those who are left behind in the pool are increasingly unable to refinance their mortgages. This burnout effect is modelled by the extra term $t^p$, $\beta_2 < 0$.

The coefficients $\gamma$, $p$, $\beta_1$ and $\beta_2$ were chosen in to give the best “fit” of the predicted balances to the observed balances. Coefficients were chosen in a Gaussian manner to minimise the sum of squared errors. The problems in estimation are considerable, due to the fact that a single observation on a particular date has information concerning multiple survival factors and optimisation results proved to be sensitive to starting conditions.

Using data from the inception of each trust to April 1994, it was found that the interest-insensitive base function had prepayments which were very low. The high level of prepayments observed must then be due to interest-rate-sensitive factors.

Figure 2 shows the base-line function for Low-start mortgage prepayments. Two distinct sets of behaviour appear for Trusts 2 and 3 and Trusts 6-15. A 25 per cent PSA model is shown for comparison. A similar picture of very low unsystematic prepayments is shown in Figure 3 for Affordable prepayments.

The list of values for the coefficients is listed in the Appendix. The interaction of the coefficients enables the model to assume similar shapes of the fitted balances for each trust for different sets of parameter values. It is difficult to interpret the results as many sets of coefficient values may result in the same shape.

We can view the observed vs predicted balances in the mortgage pools of each trust. As an example, Figure 4 shows the log of actual and fitted mortgage balances for Trust No. 3 for Low-start mortgages.

Coefficients are $\gamma = 0.01639$, $p = 2.09124$, $\beta_1 = 0.10517$ and $\beta_2 = 0$. 
The model's results must be viewed with caution because it assumes that historical behaviour will continue into the future. FANMACs were issued just before a dramatic decline in interest rates occurred and this skews the predicted prepayment rates.

**USING THE MODEL IN PRICING FANMACS**

The model can be used in an interest-rate stochastic simulator. One such simulator developed by the author projects "arbitrage-free" paths of interest rates. As FANMACs are issued by the NSW state government, the input yield curve and volatility structure of NSW T-Corp bonds are used. If working in monthly steps, at each month the prepayment model will predict the amount of prepayments due to the level of the interest rate.

**APPENDIX**

FANMAC prepayment model coefficients. The prepayment model estimated is: \( SMM_t = 1(t) \pi(t) \)

where \( \pi(t) = \exp \left( \beta_1 (\text{mortgage rate} - r) t + \beta_2 \right) \) for \( r < \text{mortgage rate} \). Otherwise, \( \pi(t) = 1 \).

<table>
<thead>
<tr>
<th>Trust</th>
<th>Low-starts</th>
<th>Affordables</th>
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<tbody>
<tr>
<td>( \gamma )</td>
<td>( p )</td>
<td>( \beta_1 )</td>
</tr>
<tr>
<td>2</td>
<td>0.01639</td>
<td>2.09112</td>
</tr>
<tr>
<td>3</td>
<td>0.01639</td>
<td>2.09112</td>
</tr>
<tr>
<td>6</td>
<td>0.01484</td>
<td>2.09112</td>
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<tr>
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<td>24</td>
<td>0.01484</td>
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These prepayments will be made at the next coupon payment and then the series of cashflows can be valued. Different sets of cashflows will arise on each path. After simulating a number of paths the expected value of the FANMAC bond can be estimated along with duration and convexity risk statistics.

For the purpose of illustration, using the model in estimating the price of the Trust No. 3 FANMAC bond with a yield curve at the close of 14/9/94 gives the results:

<table>
<thead>
<tr>
<th>Price</th>
<th>Duration</th>
<th>Convexity</th>
<th>Margin above T-Corp</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.4255</td>
<td>0.4378</td>
<td>-0.0518</td>
<td>2.073%</td>
</tr>
</tbody>
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Trust No. 3 has a reset date of 31/12/97. The results show that the high prepayments cause the bond to have a very short duration. The convexity is negative because prepayments accelerate when interest rates rise. This means the bond's value falls at an increasing rate, producing severe capital losses for investors when interest rates rise; in the past three years in Australia this is indeed what has happened. The margin above T-Corp is what the model estimates the yield on FANMAC Trust No. 3 must be to compensate the investor for the risk of prepayments.

**REFERENCES**
