HEDGING BONDS: THE CRUCIAL FACTORS

Three Insights on Tomorrow’s Yield Curve

The adequacy of models for the pricing and hedging of interest-dependent securities may depend on the number of factors driving interest rates, reports MICHAEL SHERRIS.

He explains why hedging requires a multi-factor model.

Modelling interest rates for the purposes of pricing and hedging of interest-rate-dependent securities has been a topic of much interest in recent years. For computational reasons, a single-factor interest-rate model is often used for pricing and hedging, although an increasing number of practitioners are developing multi-factor models.

Although for pricing purposes a one-factor interest-rate model might be sufficient to determine an adequate arbitrage-free pricing model, this will not be adequate for hedging purposes if the yield curve is driven by more than one factor.

Studies have been made of the interest-rate risk factors in international bond markets, including those of the US, Denmark and Italy. These studies demonstrate both similarities and differences in the various bond markets. It appears that three factors explain the major portion of yield-curve changes but the relative importance of each of these factors differs between markets.

Table 1 provides summary statistics of secondary-market yields to maturity for 13-week treasury notes and two-year, five-year and ten-year treasury bonds for the period January 1972 to October 1994. The bond yields are determined by the Reserve Bank of Australia on the basis of reported secondary-market transactions.

Yields to maturity are used in this study. It would be more theoretically correct to use spot or forward yields determined from these yields to maturity; however, the results of this study were found to be consistent with studies of other bond markets based on spot yields.

The data is monthly in the form of yields as per cent per annum. Over this period yield curves were generally upward-sloping, although there were periods when the curve was inverse. Short-term yields were more volatile than long-term yields. From mid-1982 the yield curve appears to be more volatile. This is assumed to be related to the introduction of the tender system for treasury bonds and the development of a more active secondary market.

The correlation between changes in yields of different maturities provides important information about the factors that explain the changes in the yield.

Table 1: Australian government security yields Jan 1972 – Oct 1994

<table>
<thead>
<tr>
<th>Security</th>
<th>Mean</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treasury note</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 year bond</td>
<td>10.906</td>
<td>10.750</td>
<td>2.842</td>
<td>16.400</td>
<td>5.190</td>
</tr>
<tr>
<td>10 year bond</td>
<td>10.630</td>
<td>10.240</td>
<td>2.874</td>
<td>16.400</td>
<td>5.690</td>
</tr>
</tbody>
</table>

*Note that the 13 week Treasury note, 2 and 10 year bond data include observations from July 1969 – October 1994.

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curve. For example, if the changes in yields were to be perfectly correlated, then only one factor would be required to explain yield curve changes. If there were no correlation between yields of different maturities, then a separate factor would be required for each maturity.

A more parsimonious model of yield-curve changes can be developed by determining the smallest number of factors required to explain yield curve changes. This will allow for more efficient computation of prices and hedge statistics for multiple-factor models.

Table 2 summarises the correlations between the bond yields to maturity for the data period. The correlation structure is closer to that reported for the US bond market than it is for other bond markets such as Italy. Given these correlations, it would be a surprising result if a single-factor interest-rate model, which assumes all yields are perfectly correlated, were found to be an adequate basis for explaining the future evolution of interest rates.

### Table 2: Correlations of changes in yields to maturity January 1972 - October 1994

<table>
<thead>
<tr>
<th>Maturity</th>
<th>13 weeks</th>
<th>2 years</th>
<th>5 years</th>
<th>10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>13 weeks</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 years</td>
<td>0.649</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 years</td>
<td>0.560</td>
<td>0.936</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>10 years</td>
<td>0.503</td>
<td>0.853</td>
<td>0.929</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### Factor Analysis of Yields

In order to determine the number of factors required to explain yield curve changes, a multivariate factor analysis technique is used. A factor analysis of yield curve changes estimates the following relationship for each maturity for n factors:

\[ \Delta \gamma_{mt} = \sum_{j=1}^{n} \beta_{jm} F_{jt} + \varepsilon_{mt} \]

where

- \( \Delta \gamma_{mt} \) is the change in the yield to maturity for maturity m at time t;
- \( F_{jt} \) is the value of the jth independent (random) factor at time t;
- \( \beta_{jm} \) is the factor loading for the jth factor for maturity m;
- \( \varepsilon_{mt} \) is an error term representing the variability unique to maturity m not explained by the n factors.

The factor loadings were estimated using the correlation matrix and principal component factor analysis. Three factors were assumed, since three factors have been found to explain almost all of the variability in yield-curve changes in overseas studies. The factor loadings are given in Table 3.

These factors can be interpreted as explaining different types of change in the shape of the yield curve. The first factor affects yields at all maturities by a similar amount and in the same direction. This factor can be interpreted as a parallel shift factor for this reason. The changes are not exactly parallel since the effect at the short maturity is less than at the medium to long maturities. This factor explains as much as 81% of yield-curve changes over the period of study.

The second factor has an opposite effect on the short and long yields. This factor can be interpreted as a "slope" factor, since it changes the slope of the yield curve. This second factor explains about 15% of yield-curve changes.

The third factor has a negative effect on
medium yields and a positive effect on short and long-term yields. For this reason this factor can be interpreted as a "curvature" factor. The third factor explains about 3% of yield-curve changes. In total these three factors explain more than 99% of the yield-curve changes in the data.

Compared with overseas studies, the slope and curvature factors appear to be more important in explaining the variance of yield-curve changes in the Australian bond market. A single-factor model would only be expected to replicate around 83% of yield-curve changes.

IMPLICATIONS FOR PRICING AND HEDGING

For pricing purposes, a single-factor model is commonly used. Such an approach allows efficient computation of the value of interest-dependent cashflows. Multi-factor models are often implemented using simulation and computation time is an important issue in these cases.

These models are fitted to the current yield curve and poor performance from using too few factors is often compensated for by increasing the number of parameters that are fitted for pricing purposes. Although this can be satisfactory for pricing, the above results indicate that it is not going to be satisfactory for hedging or immunisation of interest-dependent cashflows.

Since the second and third factors appear to explain as much as 15-18% of changes in yield-curve data used, portfolios constructed for immunising a set of liability cashflows using a one-factor parallel-shift model will fail to immunise against a significant proportion of yield-curve shifts. For this reason it is considered essential that multi-factor models be used for immunisation of liability cashflows using Australian government bonds.

Two or three-factor models will be adequate and there is no need to have a separate factor for each key rate yield to maturity because of the inherent correlation between changes in these yields. A similar result holds for other bond markets, although the importance of the second and third factors appears to vary from one market to another.

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