Unwarranted estimates

Biases in the valuation of convertible bonds and warrants

Convertible bonds are usually valued as the sum of two components—a “straight bond” and a warrant. JOE CHEUNG and ALASTAIR MARSDEN review issues related to the valuation of the warrant component. These issues involve both theoretical and procedural matters which are not always addressed in valuation reports.

A convertible bond is a bond which the holder is able to convert into a number of the issuer’s shares. A “straight bond”—without the conversion option—can be valued by discounting coupons and the principal amount using zero-coupon yields. The option to convert the bond into shares is similar to stock options, except that the firm issues new shares when the bond is converted. Therefore, technically, the option to convert is in effect an embedded warrant—a security which allows holders to buy newly-issued shares at a predetermined price.

Despite the conversion option being an embedded warrant, many analysts value it as a stock option using the Black-Scholes formula (the “simplified Black-Scholes approach”). Although this simplification can be justified in many circumstances, it can sometimes lead to significant biases. On the other hand, even when the conversion option is valued as a warrant, valuation procedures differ among analysts and commonly used procedures often result in even worse biases than those of the simplified Black-Scholes approach.

WARRANT VALUATION

In discussing alternative models or approaches that are used to value warrants, it is assumed the warrants are “European”, with the holder having no right of early exercise to buy new shares in the firm. It is also assumed the Black-Scholes model is a valid model to value European stock options.1 The Black-Scholes model is extensively used in practice by analysts.

Dilution effect

Galai and Schneller (1978) highlight the key distinction between warrants and call options. Unlike call options, which are “side-bets” between buyers and sellers, warrants call for the issue of new shares by the firm. It follows that the underlying shares will be diluted when warrants are exercised. Galai and Schneller derive the result that for an all-equity firm, the value of a warrant is given by:

\[ W = C(S, \sigma_S, k, r, t) \times \frac{N}{N+n} \]  

where
- \( W \) = the warrant price;
- \( C(\ldots) \) = Black-Scholes formula for a call option;
- \( S \) = share price of an otherwise identical firm which does not issue warrants;
- \( \sigma_S \) = volatility of the rate of return on \( S \);
- \( k \) = exercise price;
- \( r \) = risk-free rate;
- \( t \) = time to expiration;
- \( N \) = the number of existing shares in the firm; and
- \( n \) = the number of new shares if all warrants are exercised.

The dilution adjustment factor \( \frac{N}{N+n} \) reflects the fact that a warrant is less valuable than the corresponding call, since the share price will be diluted...
when warrant-holders exercise their right to buy into the firm at a price below the market.

It is clear that in theory the simplified Black-Scholes approach, which ignores the dilution adjustment factor, will overestimate $W$, assuming that all five parameters required by the Black-Scholes formula relating to the hypothetical identical firm are estimated correctly. In practice, however, two of these five parameters - the value and the volatility of the hypothetical underlying share - are unobservable. The hypothetical share is the share of a firm which has no outstanding warrants. The value of the hypothetical share is unobservable because the observed share price is affected by the presence of warrants.

The bias for the resulting value under the simplified Black-Scholes approach varies depending on the estimation procedures.

**Estimation procedures**

Equation (1) can be rewritten as:

$$ W = C(V/N, \sigma_s, k, r, t) \times N/(N+n) \quad (2) $$

where

- $V$ = value of the hypothetical firm;
- $\sigma_s$ = volatility of the rate of return on $V/N$;
- $C(\ldots)$ = Black-Scholes formula for a call option; and $k$, $r$, $t$, $N$, $n$ are as already defined.

Although $V/N$ and $\sigma_s$ are unobservable, the stock price $S$ and its returns volatility $\sigma_s$ of the actual firm can be estimated. The theoretical relationships between $V/N$, $S$, $\sigma_s$ and $\sigma_v$ are given below:

$$ V/N = S + (n/N) \times W \quad (3) $$

and

$$ \sigma_s = \sigma_v \times \varepsilon_{S/W} \quad (4) $$

where $W$ is given by equation (2) and $\varepsilon_{S/W}$, or $(dS/S)/(dV/N)$, is the elasticity of the stock price with respect to the firm value.

Equation (3) simply states that the value of the hypothetical firm is the sum of the value of the outstanding stocks and warrants. In other words, the observable stock price will underestimate the per "share" value of the hypothetical firm; hence, other things being equal, using $S$ instead of $V/N$ in equation (2) will also underestimate the value of the warrant.

Equation (4) states that the volatility estimated using observed stock returns is equal to the volatility of the hypothetical stock return times the stock's elasticity. This elasticity measures the instantaneous percentage change in stock price for a given percentage change in firm value, and is generally less than one. From equation (4), this implies that $\sigma_s$ is generally less than $\sigma_v$. This is also intuitive. Without warrants, stock volatility $\sigma_s$ and firm volatility $\sigma_v$ are equal for an all-equity firm. Since warrants are similar to levered stock positions, $\sigma_s$ will be smaller than $\sigma_v$ when warrants are part of the firm's assets. Therefore, the estimated stock volatility will underestimate the stock volatility of the hypothetical firm, hence other things being equal, using $\sigma_s$ instead of $\sigma_v$ in equation (2) will again underestimate the value of the warrant.

From the discussion above it can be seen that given $S$ and $\sigma_s$, $V$ and $\sigma_v$ can be solved simultaneously using equations (3) and (4), where $W$ is given by equation (2). The algorithm to do this can be implemented in an ordinary spreadsheet program such as Excel. However, many practitioners in New Zealand simply use $S$ and $\sigma_v$ with or without any dilution adjustment. Some analysts estimate and use $\sigma_s$ in equation (3) only to solve for "V/N" iteratively to calculate warrant values. (Note that the "V/N" obtained here is in general different from the $V/N$ obtained under the simultaneous equations approach.) Very few analysts approach the valuation problem with the simultaneous equations framework using both equations (3) and (4).

**VALUATION BIASES**

The effects of different parameter estimation procedures on warrant valuation are summarised in Table 1.

### Table 1: Effects of different parameter estimation procedures on warrant valuation

<table>
<thead>
<tr>
<th>Parameters estimated \ model used</th>
<th>Dilution adjustment</th>
<th>No dilution adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I: $S$ and $\sigma_v$</td>
<td>Underestimate $W$</td>
<td>Indeterminate</td>
</tr>
<tr>
<td>Case II: &quot;V/N&quot; and $\sigma_s$</td>
<td>Underestimate $W$</td>
<td>Indeterminate</td>
</tr>
<tr>
<td>Case III: $V/N$ and $\sigma_v$</td>
<td>No bias</td>
<td>Overestimate $W$</td>
</tr>
</tbody>
</table>

### Table 2: Base case assumptions and parameter estimates

- $S = \$1.00$ (current stock price)
- $k = \$1.00$ (exercise price implicit in warrant)
- $r_f = 8.00\%$ p.a. (risk-free rate)
- $t = 3$ years (time to expiration date of warrant)
- $N = 1$ million (number of existing shares in the firm)
- $n = 250,000$ (number of new shares issued by the firm if all warrants are exercised)
- $\sigma_s = 25\%$ pa (volatility of the rate of return on $S$)
- The stock is expected to pay no dividends between now and the expiry date.
- Early exercise of warrant is not allowed.
Consider first the model with dilution adjustment, and the three likely cases of parameters estimation. In Case I, \( S \) and \( \sigma_s \) are used, both underestimates of the required parameters. Since the Black-Scholes formula is non-decreasing in both stock price and volatility, the warrant will be undervalued. In Case II, \( \sigma_s \) is used to estimate \( V/N \) from equation (3). Since \( \sigma_s \) is smaller than the true volatility, the resulting \( "V/N" \) estimate will understate the true value of \( V/N \) and the warrant. Case III, where \( V/N \) is obtained under the simultaneous equations framework using equations (3) and (4), is the theoretically correct case.

Now consider the model without dilution adjustment, and the same three cases of parameters estimation. Using the same logic, Cases I and II will both underestimate the Black-Scholes value \( C \) of the hypothetical call option. However, since the dilution factor \( N/(N+n) \) is always less than one, ignoring it will overestimate the warrant value. Therefore the end result is indeterminate, and requires further analyses. Case III yields the correct Black-Scholes call value, and hence an overestimate of the warrant value when no adjustment is allowed for dilution.

A simple example of the warrant valuation errors using the simplified Black-Scholes approach (Case I, no dilution adjustment) compared with the valuation approaches incorporating dilution (Cases II and III) follows. Cases II and III with no dilution adjustment are unlikely in practice, since an analyst who performs either of the estimation procedures is likely to be aware of the dilution effect.

The base case assumptions and parameter estimates used in the example are given in Table 2 and the warrant value derived under each valuation approach is summarised in Table 3.

From Table 3, the estimate under the simplified Black-Scholes approach (Case I) is close to the theoretical warrant value with the dilution adjustment (Case III). Case II, which incorporates a dilution adjustment but incorrectly uses \( \sigma_s \) instead of \( \sigma_v \), undervalues the warrant by 4.825%.

Sensitivity analyses to the assumptions and parameter estimates in Table 2 were also undertaken with warrant values compared under each valuation approach. In most cases it was found the warrant value derived under the simplified Black-Scholes approach was close to the theoretical warrant value under the simultaneous equation approach (Case III), notwithstanding changes to the risk-free rate, the time to maturity of the option, the dilution adjustment factor and \( \sigma_s \) (volatility of stock price returns). In contrast, the valuation approach with \( "V/N" \) and \( \sigma_s \) (Case II) tended to consistently undervalue the warrant by approximately 5% or more.

One important instance exists, however, where the simplified Black-Scholes approach may give warrant values significantly different from those derived through the simultaneous equation approach (ie, the theoretical model, Case III). This is where the current share price is well below the warrant exercise price or where the warrant is deep out-of-the-money.

Figure 1 plots the warrant valuation error under the base case assumptions and parameter estimates detailed in Table 2, except that the current stock price can vary from $0.40 to $1.50. It is clear that when the warrant is well out-of-the-money (ie, the current share price is 50c or less compared with the exercise price of $1.00), the simplified Black-Scholes approach significantly overvalues the warrant. Figure 1 also

| Warrant values |
|-----------------|-----------------|-----------------|-----------------|
| Valuation approach | Parameters estimated/model used | Dilution adjustment | Warrant value | % Over/under pricing relative to Case III |
| Case I: | \( S \) and \( \sigma_s \) | No dilution adjustment | 28.23c | -0.055% |
| Case II: | \( "V/N" \) and \( \sigma_s \) | With dilution adjustment | 26.88c | -4.825% |
| Case III: | \( "V/N" \) and \( \sigma_v \) | With dilution | 28.24c | 0.000% |

**Figure 1: Warrant Valuation Error**

- Case I – Simplified Black-Scholes Approach
- Case II – \("V/N" \) and \( \sigma_s \) with dilution adjustment
- Stock Price
illustrates that the valuation approach with "V/N" and σs with dilution adjustment (Case II) undervalues the warrant. The degree of undervaluation is again greater when the warrant is out-of-the-money.

SUMMARY
While valuing the straight-bond component of a convertible bond is straightforward, valuing the warrant part requires special considerations. If the dilution effect is ignored, applying the Black-Scholes formula with the observed stock price and estimated stock returns volatility will probably provide a close estimate. Ignoring dilution will bias the estimate upwards, while using the wrong inputs to the Black-Scholes formula when the conversion option is close-to-the-money or in-the-money will provide a bias in the opposite direction. The two may cancel out (Case I, no dilution adjustment).

If the dilution adjustment is applied, it should be applied with proper procedures for parameters estimation. These proper procedures are not routinely followed, and the result is that the warrant will be undervalued (Case II, with dilution adjustment).

Given that the proper procedures are not costly to implement, the model which incorporates the dilution adjustment and the simultaneous equations parameters estimation approach (Case III, with dilution adjustment) is recommended, particularly when the warrant is well out-of-the-money.

NOTES
1 The issues that are addressed in this paper will also apply to alternative option-pricing models. For an overview of alternative option pricing models and biases in the Black-Scholes model see Hull (1997).

2 For details on the Black and Scholes option-pricing formula, see Black and Scholes (1973) or most standard finance textbooks on derivative and security pricing. The Black-Scholes model can easily be modified to provide for expected stock dividends over the life of the option (see Hull 1997). For simplicity we ignore expected dividends on the stock over the life of the option/warrant.

3 For ease of exposition, it is assumed in all hypothetical illustrations that each warrant can be converted into one share.

4 See Schulz and Trautmann (1994).

5 Also see Schulz and Trautmann (1994).

6 Only with very extreme dilution factors (ie, ratios of n/N > 2) does the warrant value under the simplified Black-Scholes approach start to appreciably differ to the warrant value under the simultaneous equations approach (Case III).

REFERENCES

Hull, J.C., Options, Futures and Other Derivatives, 3rd edition, Prentice Hall.


A service for JASSA contributors

JASSA REPRINTS

Authors may now order reprints of their articles as published in JASSA.

For details of costs and quantities, contact the publisher at:
The Custom Publishing Group Pty. Ltd.
3 Montague St., Balmain NSW 2041

Phone 02 9555 1455 Fax 02 9818 4420