Watch the curve, mind the slope

The ‘other’ signals in reading fixed-interest returns

An analysis applied to the Australian fixed-interest market examines interest-rate factors affecting the yield curve. **Dinusha Peiris** reports on the significance of interest-rate level, slope and curvature to portfolio performance.

The key determinants of the performance of an Australian fixed-interest portfolio are interest-rate factors and credit factors. This article examines the interest-rate factors contributing to changes in the yield curve, as these appear to be the most influential in the Australian context. Major components of interest-rate change across the yield curve are changes in interest-rate level, slope and curvature. While the Australian market has conventionally focused on changes in interest-rate level (or the general direction of interest rates) as the key driver, this analysis suggests that changes in the slope of the yield curve and, to a lesser degree, curvature are too significant to ignore.

Previous studies of influences on the yield curve have used approaches including principal components analysis, a statistical technique used to explain the variance of interest rates across the yield curve, and multi-factor models.

Active management of a domestic fixed-interest portfolio in recent times has usually involved an emphasis on duration management, which relates purely to interest-rate direction or level. Specifically, duration positioning assumes:

- the yield curve is flat;
- changes in the yield curve are constant across all maturities, hence a parallel shift occurs; and
- the yield curve changes by very small amounts over any small interval of time.

Under these conditions, the duration of a bond will reflect its relative performance. Similarly, at the portfolio level, the duration of the portfolio reflects performance relative to a benchmark.

While duration management is acknowledged as a key driver of fixed-interest portfolio performance, its limitations suggest that changes in slope and curvature of the yield curve also need to be considered. This analysis adopts an approach recently used in the US market and applies it to the Australian market to quantify the relative contributions made by level, slope and curvature to the overall change in interest rates across the yield curve.

**QUANTITATIVE FRAMEWORK**

The approach involved fitting Equation 1 below to spot rates across the spot yield curve. The spot rate is equivalent...
to the rate on a zero-coupon bond with maturity date equal to the term of the spot rate. For the purposes of this analysis, the Australian spot rates have been estimated using spot rates implied by bond prices.

Data available for Australia limited the analysis to the period 1990-97; however, we explored the potential for any strong biases in the results due to the particular time period considered. The US analysis provided additional history as it was based on the time period 1985-94.

Equation 1

\[ Y_{n,t} = L_t + S_t \cdot n + C_t \cdot n^2 + E_{n,t} \]

where \( E \) is the error term, \( Y_{n,t} \) is the spot rate at time \( t \) for \( n \) years to maturity, \( n \) varies from 1 to 10 corresponding to maturity dates, \( t \) varies from 1 to 96, corresponding with monthly points over eight years from 1990 to 1997. Monthly data were preferred to weekly or daily data as the analysis is more concerned with establishing trends than capturing short-term variation.

The three shape parameters in Equation 1 - \( L_t \), \( S_t \), and \( C_t \) - describe the shape of the yield curve: \( L_t \) describes level (an intercept term), \( S_t \) the slope, and \( C_t \) curvature. These parameters are estimated by regression for each of the 96 months in the period.

From the estimates of \( L_t \), \( S_t \), and \( C_t \), we can construct time series of how the shape of the yield curve changes over time as represented by changes in level (change in \( L_t \)), changes in slope (change in \( S_t \)), and changes in curvature (change in \( C_t \)). Changes over 3, 6 and 12-month periods were considered. As noted, change in level represents a parallel shift in rates across the yield curve, which is the component of the overall interest rate change that can be addressed by duration management.

The next step in the quantitative framework involves relating the change in both slope and curvature to the change in interest rate level, through Equations 2 and 3. These equations postulate that change in slope and curvature are dependent on changes in level, and enable us to evaluate the expected change in slope and curvature for a given change in level. The parameters \( B_s \) and \( D_s \) (estimated via regression) provide sensitivities of slope and curvature to a change in level.

Equation 2: Change in slope explained by change in level

\[ \text{Chg in } S_{1:s} = A_s + B_s \cdot \text{Chg in } L_{1:s} + E_{1:s} \]

Equation 3: Change in curvature explained by change in level

\[ \text{Chg in } C_{1:s} = A_s + D_s \cdot \text{Chg in } L_{1:s} + E_{1:s} \]

where \( s \) = the period over which the change is evaluated (say, six months).

Finally, converting Equation 1 into change form results in Equation 4. We can explain the mathematics in Equation 4 by noting that the expected shape of the yield curve depends on three factors:

- a term which describes how the yield curve's curvature will change, yield curve's level will change;
- a term which describes how the yield curve's slope will change;
- and another term which describes how the yield curve's curvature will change.

The estimates provided by Equations 2 and 3 are substituted for the second and third terms in Equation 4. Equation 4 is the key result of interest, as it indicates how much of the total change in the yield curve can be explained by slope and curvature relative to changes in the level of interest rates.

Equation 4

\[ Y_s = (L_0 + \text{Chg in } L_s) + (S_0 + B_s \cdot \text{Chg in } L_s) \cdot n + (C_0 + D_s \cdot \text{Chg in } L_s) \cdot n^2 \]

RESULTS OF ANALYSIS

Estimates for the \( B_s \) and \( D_s \) parameters are shown in Table 1. Equations 2 and 3 have been estimated for 3-month, 6-month and 12-month change periods. Note that the estimates are fairly consistent across different change periods. By substituting these estimates into Equation 4 we can estimate the changes in slope and curvature for maturities along the yield curve for an assumed change in the level of interest rates.

<table>
<thead>
<tr>
<th>Table 1: Equation 2 and Equation 3 parameter estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time period for changes in level, slope and curvature</td>
</tr>
<tr>
<td>3-month</td>
</tr>
<tr>
<td>6-month</td>
</tr>
<tr>
<td>12-month</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2: Slope and curvature impacts on points across yield curve for given rise in rates over six months</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impacts in basis points</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Level (given)</td>
</tr>
<tr>
<td>Slope</td>
</tr>
<tr>
<td>Curvature</td>
</tr>
<tr>
<td>Overall change</td>
</tr>
</tbody>
</table>

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For example, for a given rise of, say, 100 basis points over six months the slope effect for a seven-year security will be -63.8 basis points, while change in curvature results in a +16.6 basis-point effect. Similarly, Table 2 shows the slope and curvature impacts of a 100 basis-point assumed change in level on other representative points across the yield curve.

As indicated by Equation 4, the parameters \( B_s \) and \( D_s \) drive these estimates for a given change in level. Hence, based on the estimates in Table 2 for a 100 basis-point change, we can arrive at estimates for any other given change in the level of rates. For example, for a rise of 25 basis points, take one quarter of the numbers in Table 2; for a fall in rates, reverse the signs in Table 2 and apply the appropriate scaling.

Table 2 estimates clearly suggest that slope and curvature are significant, in terms of the final impact on interest rates at points across the yield curve. The t-statistics on the \( B_s \) and \( D_s \) parameters, which were consistently highly significant for the slope variable and mostly significant for curvature, also confirmed the relevance of slope and curvature.

The estimates confirmed that a rise in interest rates is generally accompanied by a flattening in the yield curve, while a fall in rates generally leads to a steepening. On curvature, the estimates indicated that a rise in rates resulted in a decrease in “humpedness” of the yield curve (i.e., less of a tendency for curvature to decrease at longer maturities), and a fall in rates caused an increase in humpedness.

While the results point to slope and curvature being significant, the net yield changes across maturities still indicate that interest-rate direction dominates the final return outcome. This is because after translating the overall yield changes seen in Table 2 into return outcomes, the return impacts at the longer end of the yield curve are still more significant than at shorter maturities. The return translation involves allowing for duration differences across the maturities. In return terms, the estimates in Tables 1 and 2 indicate that a general rise/fall in interest rates results in outperformance by shorter/longer maturities, hence interest-rate direction still dominates.

**Potential time period biases**

One issue that may be raised is whether the results in Tables 1 and 2 are biased by the time period on which the analysis is based. This issue can be investigated by looking at the time series of changes in level, slope and curvature derived from the estimates of Equation 1. If the period is dominated by a particular trend, which may be unrepresentative of other time periods, then this should show up in the relationship between the

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**Figure 1: Changes in level and slope over period 1990-97**

**Figure 2: Changes in level and curvature over period 1990-97**
change in level and slope and the change in level and curvature over the period of analysis. Figures 1 and 2 indicate that a reasonable amount of variability was seen in the relationship between level on the one hand and both slope and curvature on the other. Hence, results reported in Tables 1 and 2 should not reflect strong biases due to the time period.

US COMPARISON
Table 3 provides the comparable Table 2 results for the US market. The US study was based on monthly data over the period 1985-94, hence provides additional history to our analysis of the Australian market. Several similarities were found between the two studies.

The estimates for the \( B_s \) and \( D_s \) parameters for both markets have the same sign across both markets (negative and positive respectively), pointing to the same flattening/steepening in the yield curve as interest rates rose/fell, while curvature increased/decreased as interest rates rose/fell.

The estimates were consistent over change intervals up to 12 months for both the US and Australia.

The magnitudes of the \( B_s \) (slope) and \( D_s \) (curvature) parameters appeared to differ somewhat – \( B_s = -0.04213 \) for the US compared with \( -0.0911 \) for Australia, and \( D_s = 0.00084 \) for the US compared with \( 0.0034 \) for Australia. However, some of the differences may be explained by the different range of maturities across the two markets; the US yield curve extends to about 30 years compared with 10 years for Australia. Hence, comparing slope and curvature impacts (in Tables 2 and 3) across “like” parts of the yield curve points to less differences, longer-dated being close to 30-year for the US and 10-year for Australia, and the mid-part of the curve being around 10-year for the US and 5-year for Australia.

CONCLUSION
This analysis indicates that while changes in interest-rate level (ie, the general direction of interest rates) still dominates relative returns on fixed-interest securities, changes in slope and curvature of the yield curve play a significant role. The quantitative approach adopted, based on a recent US study, confirmed that a rise in rates is generally accompanied by a significant flattening in the yield curve, and a fall in rates generally leads to a significant steepening. Nevertheless, even after allowing for slope and curvature effects, a rise/fall in interest rates still results in outperformance by shorter/longer dated securities across the yield curve. Findings on the Australian market were similar to those for the US.

While the results point to slope and curvature being significant, the net yield changes across maturities still indicate that interest-rate direction dominates the final return outcome.

Table 3: Slope and curvature impacts (US market) on points across yield curve for given rise in rates over six months

<table>
<thead>
<tr>
<th>Impacts in basis points</th>
<th>Representative maturities across yield curve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1yr</td>
</tr>
<tr>
<td>Level (given)</td>
<td>+100</td>
</tr>
<tr>
<td>Slope</td>
<td>-4.2</td>
</tr>
<tr>
<td>Curvature</td>
<td>+0.1</td>
</tr>
<tr>
<td>Overall change</td>
<td>+95.9</td>
</tr>
</tbody>
</table>

NOTES

The JASSA Prize
All original articles published in JASSA are eligible for the JASSA Prize, awarded annually for the article judged as making the best contribution to the securities industry. As well as the major JASSA Prize of $1,000, there are up to three merit awards, each of a Mont Blanc Meisterstuck fountain pen.