Managing SPI risk: using Value at Risk in equity futures markets

Value at Risk is a widely accepted measurement tool but as MICHAEL LIBERMAN explains, there are problems that you need to be aware of.

Value-at-Risk (VAR) has become a benchmark for risk measurement within financial institutions and fund managers, as well as corporate treasurers. Senior managers use VAR to measure the market, or the price risk of a portfolio of assets—this is the risk that the market value of a portfolio of assets will decline as a result of changes in interest rates, foreign exchange rates, equity prices, or commodity prices.

Furthermore, VAR allows an institution to measure the risk/reward characteristics of its various businesses and allocate a return on capital to each. Financial regulators use VAR to determine if an institution holds enough capital to withstand unexpected market losses. VAR has become popular over recent years as it provides senior management and central bank regulators with a single number that summarises the total market risk of a portfolio of assets.

In calculating portfolio risk, VAR methodologies attempt to capture both volatility and correlation of a portfolio of assets; these are the fundamental risk drivers used by VAR models. However, there are difficulties in applying standard VAR methodologies to a portfolio consisting of equity futures and equity futures options. There are several methods used to overcome the idiosyncrasies equity futures present but most fail to address the underlying issue. What is required is an appropriate use of the fundamental risk drivers so that a meaningful VAR measure can be derived.

VAR modelling
A VAR model attempts to impound the market risk of a portfolio into a single number. This is done by determining how much the value of a portfolio could decline by over a given period of time and with a given probability.

The important parameters used for VAR modelling are the holding period or time over which market risk is to be measured, and the confidence interval or probability at which market risk is to be measured. For example, if the holding period is one day and the given probability is 5%, then the VAR measure would be an estimate of the decline of the portfolio value that could occur with a 5% probability over the next trading day. In other words, a risk manager can be 95% confident that portfolio losses would not exceed the VAR measure over the next trading day.

Setting both risk parameters—namely the holding period and confidence interval—is discretionary. The nature of VAR is greatly affected by the choice of these parameters. A fundamental assumption of VAR models is that the portfolio’s composition does not change over the holding period. For a bank, where the portfolio is invested in highly liquid assets, a one-day holding period may be acceptable. However, a 90-day holding period may be more relevant to an investment manager who has quarterly portfolio reporting and re-balancing.

Ideally, the holding period should correspond to the longest period needed for orderly portfolio liquidation. A bank’s active trading portfolio is much easier to close out than a portfolio invested in stocks from emerging markets.

The confidence interval reflects a company’s appetite for risk. The more conservative financial regulators
enforce a 99% confidence interval whereas the more liberal hedge funds would perhaps use a 95% confidence interval or lower.

**Historical simulation**

There are several methodologies used to compute the VAR measure with large financial institutions using historical simulation to model VAR. Historical simulation is an attractive modelling technique as it is highly intuitive to both senior managers and regulators. Furthermore, historical simulation accurately reflects the probability distributions of the market variables as well as their impact on the valuation of a portfolio.

In order to implement the VAR model, a complete time series of each market variable that affects the valuation of the assets in a given portfolio needs to be collected. Using a time series of say, a common stock, in a VAR model is relatively straightforward—stock prices are readily observed in the market and can be fed directly into a VAR model.

For example, a portfolio consisting of a single stock can compute the current day’s VAR for this portfolio—having, for example a one-day holding period and a 95% confidence interval—by taking a 100-day time series of the stock price. We would then take today’s portfolio stock composition and construct a series of 100 hypothetical scenarios, whereby each scenario is a profit or loss resulting from daily rate changes in the stock price.

The first scenario would be calculated as the profit or loss resulting from the stock price appreciating or depreciating from the second to the first day in the time series. The 100th scenario would be the resultant profit or loss as the stock price varies from the 99th to 100th day. What we now have is a vector of 100 hypothetical profit and loss scenarios, given today’s portfolio composition. The VAR for the portfolio is simply the fifth worst loss in the profit and loss vector (see Chart 1). A fundamental assumption of this type of modelling is that each scenario outcome has an equal probability of occurring.

The single stock portfolio VAR in Chart 1 was easily calculated as the stock price time series that captures the variability or volatility of daily returns, which gave rise to portfolio VAR. But what about a portfolio which is composed of two or more stocks? In order to compute VAR for multi-stock portfolios we would need a time series for each stock in the portfolio.

Applying the above methodology would not only capture variability of daily returns for each stock in isolation, but would also account for the correlation between stock returns in the portfolio. A well-known result of Modern Portfolio Theory is that asset correlation has much greater weighting on portfolio risk than individual asset returns in isolation.

Historical simulation can be extended to a portfolio of stock options whereby historical stock volatility, interest rates and dividends would be additional risk factors used in the simulation. All the fundamental market risk factors that affect the valuation of a portfolio must be included in the simulation. Historical distributions of asset returns, asset return correlations and all other risk factor returns and correlations are captured in the time series to give an intuitive and meaningful VAR measure. The methodology is both simple and transparent.

**Applying VAR to equity futures and equity futures options**

Historical simulation becomes less clear when dealing with a portfolio of equity futures and equity futures options. Unlike common stock prices, futures prices need to be transformed to a useable time series. The SPI Futures contract is based on a Price Index, not an Accumulation Index, and has to make allowance for the incidence of stocks going ex-dividend before the expiry date of the contract.

Generating a time series for futures prices is not intuitive and requires a bit of thought. Take, for example, a futures contract that has 90 days to maturity and call it, say, a Dec-2002 maturity contract. This futures contract, tomorrow, will have 89 days to maturity and 88 days to maturity on the following day. After a 90-day cycle this contract will mature and roll-over and a new contract will start trading. Over a 90-day period there will be 90 futures contracts and hence 90 prices for the same futures contract, Dec-2002. Furthermore, the near month contract will experience an extensive price jump on the roll over date.

We cannot create a meaningful time series for the Dec-2002 contract as we did for the common stock. Common stock price series still have to be adjusted for dividends and capital dilutions, etc. This is because the futures prices, for the same contract,

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**CHART 1 PROFIT AND LOSS DISTRIBUTION**

- Frequency (Number of days)
- 5% of occurrences
- 95% of occurrences
- VaR Loss (95%)
- Profit & Loss Scenarios ($)
have decreasing maturities and hence represent different risk factors. The problem is exacerbated by the fact that there are six live contracts at any one time all with decreasing maturities and quarterly roll-over dates giving rise to a discontinuous price series. A meaningful time series would need to capture specific maturity risk or volatility, as well as the correlation effects for a portfolio containing multiple contracts of varying maturities. This problem can be resolved by the use of constant maturity forwards. These constant maturity forwards are simply synthetic instruments that always mature a fixed number of days from the current date. A 30-day constant maturity forward will always mature in 30 days from the reference date. A complete constant maturity term structure can be generated, having say: 30, 60, 90, 180, 270 and 360 days to maturity. Once a time series is generated, using these constant maturity points, it is a simple matter to calculate VAR for a given portfolio. Both futures price volatility and price correlation will be captured in much the same way as a portfolio containing multiple stocks.

The challenge is to convert or map the actual traded futures prices to a constant maturity term structure for each date in the time series. The mapping methodology must be consistent so that futures price volatility and correlation is correctly represented in the time series to yield a meaningful VAR measure.

SPI200 futures: fair market value

The Sydney Futures Exchange (SFE) currently trades the Share Price Index futures (SPI200) contract. The SPI200 contract is based on the S&PE/ASX200 Index, which is calculated using the market prices of the top 200 Australian companies listed on the ASX. Each index point in the SPI200 contract has a dollar value of $25 per contract.

The fair market pricing of the SPI200 futures contract is based on arbitrage-free arguments. An investor could replicate the pay-off of a SPI futures contract by borrowing cash today in order to fund the purchase of a portfolio of stocks that make up the Index—‘the funding costs are offset by any dividends received until maturity. At maturity the portfolio may be liquidated and the loan repaid; the cost of the replicating portfolio was known at inception. In theory, market forces should bring the SPI futures price in line with the replicating portfolio, or else an arbitrage opportunity would arise. Hence, the fair market value of the SPI200 contract is given in Equation 1.

\[ \text{fair price}_{\text{SPI200}} = \text{Index}_{\text{S&PE/ASX200}} \times (1 + \text{cost of funds \%} \times \text{time factor}) \]

where the cost of funds is estimated by:

\[ \text{cost of funds \%} = \text{borrowing rate \%} - \text{portfolio dividend yield \%} \]

### Table 1: Constant Maturity Forward Mapping

<table>
<thead>
<tr>
<th>Contract Name</th>
<th>Maturity Date</th>
<th>Days to Maturity</th>
<th>Futures Price</th>
<th>Days to Maturity</th>
<th>Forwards Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sep-02</td>
<td>30-Sep-02</td>
<td>60</td>
<td>3063</td>
<td>30</td>
<td>3059</td>
</tr>
<tr>
<td>Dec-02</td>
<td>19-Dec-02</td>
<td>140</td>
<td>3071</td>
<td>60</td>
<td>3063</td>
</tr>
<tr>
<td>Mar-03</td>
<td>31-Mar-03</td>
<td>242</td>
<td>3080</td>
<td>90</td>
<td>3064</td>
</tr>
<tr>
<td>Jun-03</td>
<td>19-Jun-03</td>
<td>322</td>
<td>3093</td>
<td>180</td>
<td>3069</td>
</tr>
<tr>
<td>Sep-03</td>
<td>18-Sep-03</td>
<td>413</td>
<td>3108</td>
<td>270</td>
<td>3074</td>
</tr>
<tr>
<td>Dec-03</td>
<td>18-Dec-03</td>
<td>504</td>
<td>3108</td>
<td>360</td>
<td>3080</td>
</tr>
</tbody>
</table>

*Source: Bloomberg, 1 August 2002.*
would enable him to create a synthetic forward. Fig. 1A shows the price profile of a futures strip using the first two futures contracts traded, i.e. Sept–02 and Dec–02 respectively. The construction of the SPI futures strip price profile is straightforward. All instruments that mature prior to the synthetic forward point have a flat profile, whereas a linear profile is assumed between the last two futures; the synthetic forward falls between the last futures in the strip. The price profile up to and including the Sept–02 contract maturity is flat at 3,063 as an investor can lock in the 60-day futures price by entering the Sept–02 contract. The price profile between the Sept–02 and Dec–02 dates is assumed linear given the arguments described above. The interpolated futures price at the 90-day point turns out to be 3,066.

Each segment of the futures strip can be thought of, notionally, as a sub-portfolio, having units of dollars per unit of time. The total cost of holding the futures strip is the summation of the cost of each segment or sub-portfolio. This means that the cost of holding the portfolio, in dollar terms, is simply the area under the price profile in Figure 1A. Using the same argument, a single synthetic 90-day forward contract can be formed having a flat price profile (see Figure 1B). The price of a flat synthetic forward is not known but can be implied by equating the cost of the two portfolios, i.e. the replicating strip and the synthetic forward. The cost of each portfolio is simply the area under each graph (Equation 2).

The unknown can be solved to give the 90-day forward price of 3,064 index points. The same result would be derived if index points were converted to their dollar value equivalent of $25 per index point.

Using the same arguments the 180-day forward price can be deduced—see Figure 2A and Figure 2B. The strategy would be an extension to the previous strategy implemented. After 60 days the investor would liquidate the Sept–02 contract and initiate the Dec–02 contract maturing in 80 days (140 minus 60). The Dec–02 contract would be liquidated at the 140-day point. The 180-day point gives an interpolated price of 3,074 falling between the Dec–02 and Mar–03 contact maturity dates. Once again, equating the cost of the two portfolios implies the price of a synthetic 180-day constant maturity forward to be 3,069 index points (Equation 3).

The entire constant maturity forward term structure is determined using this method. The above process can be formalised into Equation 4.

The terms \( p \) and \( t \) represent the futures prices and their maturity dates respectively; there are \( k \) futures in the replicating strip such that the constant maturity forward matures after the \( k \)th future.

The above equation uses exponential interpolation rather than linear interpolation used in the example.

There are other techniques that could be employed to generate the synthetic forward prices. The advantage of using the entire futures strip in the mapping process is that both volatility and correlation of quoted futures prices are captured consistently and the resultant

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**EQUATIONS 2 & 3**

\[
60 \times 3,063 + \frac{1}{2} \times (90 - 60) \times (3,063 + 3,066) = 90 \times ?
\]

\[
60 \times 3,063 + (140 - 60) \times 3,071 + \frac{1}{2} \times (180 - 140) \times (3,074 + 3,071) = 180 \times ?
\]
price distributions are representative of the market.

**SPI risk in practice**

Now let's look at a practical example. The current SPI forward curve, as of August 1, 2002, is upward sloping. A SPI trader is expecting the forward curve to flatten over time and wishes to profit from his expectations. The trader executes a calendar spread and buys one Sept–02 contract and sells one June–03 contract. How can the trader be 95% confident that he will not lose more than X dollars over the next trading day; the variable X represents the trader's portfolio VAR.

Using the above methodology, a 100-day historical time series is generated at the 60 and 322-day points, i.e. the contracts’ maturity dates. Using the time series 15–Mar–02 to 1–Aug–2002 (100 working days), 100 hypothetical profit or loss scenarios are generated using the day-over-day rate changes applied to the current portfolio composition. The hypothetical profit/loss vector is then sorted in ascending order from 1 to 100. The profit/loss vector is then sorted in ascending order from 1 to 100. The hypothetical profit/loss vector is then sorted in ascending order from 1 to 100. The fifth worst loss gives the portfolio VAR.

Table 2 below shows the five worst losses for each contract in isolation as well as for the diversified portfolio:

The 95% VAR for the Sept–02 and June–03 contracts in isolation is $1,064 and $780 respectively, though the total portfolio VAR is $71 per contract. This is expected as the two individual contacts are highly correlated and are positioned to offset one another. An important observation is that the loss dates are not aligned for each asset. When one contract shows a gain for a given day, the second shows a loss and vice versa.

Back-testing is an important tool used to assess the appropriateness of a given VAR model. The actual losses for the above portfolio should breach VAR on no more than five sample days in every 100. If such violations occurred on more instances than expected then the VAR methodology would need to be reviewed.

**Discussion**

VAR modelling using historical simulation requires a complete time series of each fundamental risk factor that affects the valuation of a portfolio. Each risk factor must have a complete time series whereby historical prices are represented at constant maturity points. The hypothetical profit/loss vector is then sorted in ascending order from 1 to 100. The fifth worst loss gives the portfolio VAR.

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**Discussion**

VAR modelling using historical simulation requires a complete time series of each fundamental risk factor that affects the valuation of a portfolio. Each risk factor must have a complete time series whereby historical prices are represented at constant maturity points. Forming a replicating portfolio for each date in the time series can be used to convert market quoted futures prices to constant maturity forward prices. This method is applicable to equity derivatives whose underlying risk factor is a futures price. This method is equally as applicable to stock futures and stock futures options as it is to Index instruments such as SPI futures and SPI futures options. However, this method is not applicable to other investment assets and cannot be used to price commodity futures.

Interest rate futures products such as bank bill and bond futures do not require the constant forward price conversion used for equity futures. Interest rate products can be bootstrapped, i.e. replicated by a strip of zero coupon bonds. This procedure enables all interest rate instruments to be priced off a zero curve or discount curve function. These functions are classed by credit rating and facilitate consistent pricing of all interest rate instruments at constant maturity forward points.

Foreign exchange futures can be represented as three separate instruments: one spot foreign exchange instrument and two money market instruments. The two money market legs can be bootstrapped in their respective economy as any other interest rate product and subsequently mapped into constant maturity forwards. Therefore, neither foreign exchange futures nor interest rate futures require constant maturity forward conversion.

Commodity futures do not follow the arbitrage-free pricing models that are generally observed for investment assets—these are highly liquid assets that are traded purely for investment

**Continued on page 48**
**Superanunnation**

Is superannuation really taxed concessionally? By David Knox. Spring. A case is made to show that government taxation on superannuation is a disincentive to saving.

**Super funds and proxy voting** by Geof Stapledon. Winter. The new ASX Corporate Governance guidelines is putting pressure on funds managers to exercise their proxies.

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**Valuation**

Valuation bias in projects with tax losses by Joe Cheung and Alastair Marsden. Winter. Tax losses on projects often decide their viability and the author looks at a number of methods to measure them.

**Authors index**

Alexander, James (with Jamie Krasowski). The value of adding corporate bonds to Australian fixed interest portfolios. Summer.

Alles, Lakshman. Accounting for employee stock options. Autumn.


Bienkowski, Nik. A golden rule in risk management. Spring.

Brooks, Robert (with Robert Faff, David Hillier and Joseph Hillier). Do stock markets react to the re-rating of sovereign risk? Summer.

Canil, Jean (with Bruce Rosser). Executive options: Don’t throw baby out with the bathwater. Winter.


Cheung Joe (with Alastair Marsden). Valuation bias in projects with tax losses. Winter.


Gallagher, David (with Andrew Looi). Are active managers more successful? Autumn.


Luciano, Robert. EBITDA as an indicator of earnings quality. Autumn.

Nguyen The Tho (with Ian Eddie). Winter. The Vietnam securities market.

Romijn, Simon. Australian fixed interest—the sure nickel versus the uncertain dollar. Autumn.


Stuart, Marc (with Justin Guest and Fred Wellington. Summer. The role of emissions trading in Asian clean energy finance.

Thi Tu Le Duong (with Neil Hartnett and Joseph Winsen). Understanding implied volatility and market stress in equity and fixed interest markets. Summer.

Utharntharm, Direk. Sector diversification, home-country bias and global investments. Winter.

Continued from page 45

purposes such as foreign exchange, interest rate and equity products. Investment asset futures, generally, can be replicated with a portfolio consisting of the underlying investment asset and a risk-free bond. Most commodities are said to be consumption assets which have forward prices that are driven by supply and demand for the physical commodity.

**CONCLUSION**

Constant maturity mapping is an essential requirement for building sound VAR models. Various investment and consumption type assets have unique attributes.

Therefore constant maturity mapping techniques cannot be used homogenously across all asset classes. Each asset’s idiosyncrasies must be addressed so that a meaningful VAR measure can be derived.

An important reality check for calculating VAR is back-testing. This testing determines how well VAR estimates would have performed in the past and whether the methodology is suspect and needs to be reviewed.

**REFERENCES**


