Investor expectations and beta risk

Investors have short memories and tend to concentrate on today’s gains rather than yesterday’s losses. Understanding what drives investment decisions is never easy and ADAM CLEMENTS and MICHAEL DREW use an adaptative expectations approach to make some empirical observations.

How are asset prices and future pay-offs related? The modern paradigm, led by the classic studies of Sharpe (1964) and Linter (1965), suggests that the value of an asset is dependent upon investor expectations. The Sharpe-Linter Capital Asset Pricing Model (CAPM) showed that if investors had homogenous expectations (and optimally hold mean-variance efficient portfolios in the absence of market frictions), the portfolio of all invested wealth, or the market portfolio, will itself be the mean-variance efficient portfolio (Hawawini 1984, Campbell, Lo and MacKinlay 1997).

In essence, the CAPM is a statement about expectations. Investors are viewed as self-seeking, risk-averse price takers, with homogeneity of expectations regarding asset returns. However, little is understood of the process by which investors formulate expectations or what causes investors to act on them.

We argue that the CAPM assumption of consistency of investor expectations across the marketplace may be interpreted in one of two ways. Initially, it can be asserted that all market participants do in fact hold one view (homogenous expectations) of the investment environment as postulated by the CAPM. As no difference of future pay-offs is observed, no differences across opinions of current asset value will be apparent.

However, if differences are observed, a mechanism must exist that permits investors to act in response to these differences. Thus the marketplace for financial assets, freely accessible to all investors, allows transactions to be conducted reflecting divergence of investor opinions. Therefore, the market-clearing price will be a true reflection of aggregate opinion.

This paper considers the issue of the process by which investors form expectations and the implications for the received asset pricing theory—the CAPM.

Motivation for much research in the modern paradigm has been concerned with explaining ex-post returns on both individual stocks and portfolios. In a similar vein to prior research, the motivation for this study is to refine the estimation of beta ($\beta$) or systematic risk. This study is similar to that of Brooks and Faff (1997) in that it attempts to better account for future beta and returns utilising past data. While Brooks and Faff (1997) do improve on the forecast of future beta, they employ ex-post adjustments to standard estimates. This paper provides an alternative to this approach, through an examination of investor expectations.

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SYSTEMATIC RISK AND ADAPTIVE EXPECTATIONS

According to the CAPM, expected returns are a function of the systematic risk of an asset, with expected beta measuring systematic risk. The units of systematic risk that the beta measures, along with the expected market risk premium and the expected return on the risk-free asset, determine the total return on a risky asset.

Along with the market return, systematic risk is the most significant input for determining security returns. Bos and Newbold (1984), Collins, Ledolter and Rayburn (1984) and Brailsford, Faff and Oliver (1997) find, however, that beta estimates are time varying. This variation is attributed to both market-wide factors along with firm specific events such as new investments undertaken by the firms.3

It is clear the determinants of total return vary randomly through time, which may result in discrepancies between the expectation of returns and actual observed returns. As this issue is difficult to control at the individual stock level, this study considers the expected returns on portfolios. This effectively negates any consistent over- or under-expectation of returns due to firm specific characteristics.

It is clear that expectations on the market portfolio and the individual asset are important when formulating predictions of systematic risk of an asset in the subsequent period. While the CAPM explicitly assumes no particular process under which expectations are formed, standard estimation techniques imply certain behavioural patterns.

Traditionally, estimation of the market model (MM) employs standard Ordinary Least Square (OLS) regression, where the slope parameter is defined as follows:

\[
\beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} = \frac{\text{E}(R_i - \bar{R})(R_m - \bar{R}_m)}{\text{Var}(R_m)} 
\]

All pairs of observations, \(R_i\) and \(R_m\), are given equal weighting in the calculation of the coefficient. Such an estimation technique implies that investors place equal importance on all previous observations in the series when forming their expectations.

However, empirical evidence from the field of psychology suggests that such an implicit assumption invoked under the CAPM is inconsistent with behavioural patterns.4

Elliott and Anderson (1995) investigated the effect of a memory decay process on the performance of individuals to predict the outcomes of a series of changing categories. Real world categories (of which a series of asset returns is an example) undergo gradual and systematic changes over time.

Elliott and Anderson (1995) found that participants successfully adjust to category change, revealing a lingering and cumulative effect of past observation; thus performance is best modelled by incorporating a memory decay process.5

The contribution of Elliott and Anderson (1995) has important implications for traditional estimation methods of the CAPM. In the context of this study, we employ a model of adaptive expectations (AE), described as an exponentially weighted, moving average forecast, to account for the process by which investor expectations may adapt to changing conditions.

We augment the standard AE model for the CAPM setting in the form:6

\[
R_i = \beta_0 + \beta_1 R_{m*} + \mu_t
\]  

[2]

Where the value of \(R_i\) is dependent on the expected value of \(R_{m*}\) represented as \(R_{m*}\). The \(\beta_1\) reflects the expected systematic risk of the \(i^{\text{th}}\) asset, accounting for the relationship between the return on the asset (along with its expected return, assuming rational expectations) and the expected return on the market portfolio.

As \(R_{m*}\) is an expectation variable (and is therefore not observable), the following process by which expectations change across time is assumed:

\[
R_{m*} = \gamma (R_m - R_{m-1*}) + \mu_t
\]

[3]

Representing the adaptive expectations hypothesis in the form of Equation [3], where \(\gamma\), such that \(0 \leq \gamma \leq 1\), is known as the coefficient of expectations. AE predicts investors alter expectations between periods by a fraction of \(\gamma\), of the difference between the outcome of the market return and its expected return. Equivalently:

\[
R_{m*} = \gamma R_{m} + (1 - \gamma) R_{m-1*}
\]

[4]

Substituting this result into the original expectations form of the CAPM, the AE model is defined as:

\[
R_i = \gamma \beta_0 + \gamma \beta_1 R_{m*} + (1 - \gamma) R_{m-1*} + \mu_t
\]

[5]

Where \(\mu_t = \mu_t - (1 - \gamma) \mu_{t-1}\). Once an estimate of \(\gamma\) is obtained from the coefficient of \(R_{m-1*}\) variable, values for \(\beta_1\) are found by dividing the \(R_{m*}\) coefficient by \(\gamma\).7 Thus an AE approach facilitates the estimation of investors’ expectations of systematic risk and returns, allowing for behaviour consistent with that of previous behavioural studies.

The AE conception of the CAPM permits investors to adapt expectations of future outcomes in response to forecasting errors from previous periods. The dynamism of the model allows parameter values to efficiently adjust to shocks entering the system. We argue that this model better reflects the way in which investors quickly incorporate new information into expectations, prices and returns. Consistent with this theme, we suggest that the MM places excessive weight on earlier observations from which the effects of any information would have dissipated from the system. Conversely, standard approaches appear to discount the effects of very recent experiences on expectations and model parameters and do not fully incorporate their importance on the pricing of risky assets.

DATA COLLECTION

The data used in this study has been gathered from a number of sources. Information pertaining to individual share prices and market indices was obtained from the Centre for Research in Finance (CRIF) as collected by the Australian Graduate School of Management (AGSM) at the University of New
South Wales. Data regarding return on the risk-free asset was drawn from the Australian Bureau of Statistics time series service. Information on the trading frequency of selected stocks was sourced from the AGSM risk measurement service.

Prior to shares being included in the final estimation stage of the study, each had to meet certain criteria. Each stock had to have been listed on the Australian Stock Exchange (ASX) for the entire period from January 1992 to December 1997. If stocks were deemed to be infrequently traded they were excluded from the study. Listing history was determined from the CRIF file as it contained list and de-list dates for all securities listed on the ASX.

The AGSM risk measurement service supplied a measure indicating the frequency of which the trading in a share occurs. Stocks were excluded if this measure (termed the LM Statistic) was less than 0.05, indicating that adjustments to systematic risk estimates were necessary to account for thin trading.

This process resulted in 300 stocks being included in the sample. These stocks were then randomly combined into 20 portfolios each containing 15 securities to ensure sufficient diversification within each portfolio. Portfolios were formed on the basis of equal investment weights. Returns on individual securities were then calculated:

\[ R_t = \left( P_t + D_t + C_t - P_{t-1} \right) / P_t \]  

Where \( P_t \) and \( P_{t-1} \) are the share prices in the current and previous periods respectively. Dividends paid in the current period are represented by \( D_t \) while \( C_t \) indicates capitalisation changes. Portfolio returns were calculated as the simple average of the individual security returns as the portfolios were constructed with equal investment weights.

Market proxy returns were obtained from the value-weighted market index also contained in the CRIF file. Following Brailsford et al. (1997), a value-weighted index measure was preferred to an equally weighted index of market returns as it is more consistent with the true market portfolio.

The observation period from January 1992 through December 1997 was divided into two sub-periods. January 1992 through December 1995 is dedicated for the estimation of systematic risk of each perspective portfolio. Tests of estimate precision were based on both ex-post explanatory power (along with forecasting capability) for the period January 1996 through December 1997.

**ANALYSIS**

**A. Market Model**

Summary statistics are provided in Table 1 from the estimation of the MM:

\[ R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_i \]  

Even at the 10 per cent level of significance, systematic risk estimates for Portfolios 1, 3 and 7 were not statistically different from zero. Therefore, returns on these portfolios do not appear to be related to returns on the market index. Assuming the 10 per cent level of significance, all remaining portfolios yield significant results. The explanatory power of the model, captured by the \( R^2 \) statistic, range from negligible values to around 0.18, with, as expected, very poor results arising from the portfolios with insignificant beta estimates.

**B. Adaptive Expectations**

Table 2 presents equivalent statistics from estimation under the AE approach of the form in Equation [5]. Results in the Constant, Market and Lag columns represents \( \gamma \beta_{i0} \), \( \gamma \beta_{i1} \) and \( (1 - \gamma) \) respectively.

The AE procedure only yields two insignificant market coefficients, portfolios 3 and 7, which were also both insignificant under the MM. Coefficients applying to past return series were observed to be highly significant in all cases where the market coefficient was also significant. The \( R^2 \) measures once again commence from very low values. However, across the board, the AE model appears to have dramatically increased the explanatory power across the portfolios.

As the AE approach contains a lag of the dependent variable as an explanatory variable, the incremental explanatory power of this term above and beyond that of the MM is formally considered through the calculation of the \( F \)-statistic for each portfolio:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Alpha (( \alpha ))</th>
<th>t-statistic</th>
<th>Beta (( \beta ))</th>
<th>t-statistic</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0179</td>
<td>2.059</td>
<td>0.1462</td>
<td>0.845</td>
<td>0.0156</td>
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<td>2</td>
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<td>2.809</td>
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<td>3</td>
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<td>2.488</td>
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<td>0.0020</td>
</tr>
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<td>4</td>
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<td>0.3936</td>
<td>2.306</td>
<td>0.1057</td>
</tr>
<tr>
<td>5</td>
<td>0.0030</td>
<td>2.178</td>
<td>0.4172</td>
<td>1.994</td>
<td>0.0812</td>
</tr>
<tr>
<td>6</td>
<td>0.0190</td>
<td>2.284</td>
<td>0.4236</td>
<td>2.577</td>
<td>0.1286</td>
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<tr>
<td>7</td>
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<td>-0.6993</td>
<td>-2.235</td>
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<td>0.0745</td>
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<td>0.0928</td>
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<td>0.0736</td>
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<tr>
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<td>0.5834</td>
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</table>
Table 3 reports $R^2_{old}$ and $R^2_{new}$ statistics from both MM and AE approaches respectively. Probability values for the $F$-statistic are provided in the final column.

Again, excluding the insignificant portfolios 3 and 7, application of the AE results in significant increases in explanatory power in all of the remaining portfolios. It appears to be important to consider an exponentially weighted series of past returns when considering current asset returns.

Moreover, the results support the notion that recent experiences are of greater importance to investors in the formation of expectations in that they continually adapt expectations to errors from the past. To investigate this idea further, the next section examines the forecasting capabilities of the AE measure compared to the traditional MM approach.

### C. Forecasting

Forecast accuracy is determined by comparing estimated returns to that of the actual observed return, calculating the squared error (SQE) of the forecast:

$$SQE = (R_{it} - \hat{R}_{it})^2$$

SQEs were calculated for all estimates obtained under the MM and AE approaches. Mean Squared Errors (MSE) were then computed across all portfolios for each time period. Table 4 (overleaf) outlines the MSEs for both approaches for each of the 24 months over the period January 1996 through December 1997. These estimates are based on the parameters estimated during the previous four-year period. A score of one in the final column in Table 4 indicates superior performance obtained under AE.

An analysis of MSE found that AE produced more accurate forecasts in 15 of the 24 months (reflected in a smaller MSE for that month). The best performance of the AE approach was recorded in the first 12 months, with nine out the 12 months AE producing superior results to MM. Over the next 12-month period, the results are split evenly (six out of 12 months) for both the AE and MM approaches.
**Conclusion**

Improved explanatory power of current returns based on the inclusion of the past series of returns supports the notion that investors, when forming expectations, exhibit a memory decay process. Greater importance appears to be placed on most recent experiences in comparison to those much further back in time.

The evidence provided in this paper found that the AE approach provided superior forecasts in the short term than its traditional MM counterpart. Invoking the assumption that, on average, investor expectations will be realised, AE would appear to be a valid account of the process by which investors form expectations. The fact that AE provided superior performance during the short term is to be expected with such a dynamic model.

This idea is further supported by the forecasting abilities of the two approaches. AE accounted for the immediate future (up to 12 months) with more accuracy than the MM approach. Very recent information appears important in explaining short-term changes in returns. The added emphasis placed on the more recent observations in the estimation of the AE is the contributing factor to its superior performance in the short term. The relevance of this recent information decreases when attempting to explain returns that are further out in time.$^{11}$

Beyond 12 months into the future, no significant pattern is observed between the MM and AE methods. As the forecasts were based on parameters estimated from data that is more than a year old, the structure of the system had apparently changed. No single method was consistently superior which would lead to the conclusion that relationships observed in the past were no longer strictly valid.$^{12}$

Distributed lag models, such as AE, allow greater importance to be placed on more recent observations, reflecting the fact that expectations continually adapt. This is in stark contrast to the equally weighted OLS technique. The results of this paper support the notion that earlier events are of less importance in explaining asset prices, in comparison to more recent experiences.

Further research leading from these results may take two paths. The information set that investors consider when forming expectations may be investigated. It is possible that a wider information set, if available, may be more representative of the true considerations of investors. Second, alternative econometric techniques may also be valid accounts for the behaviour patterns of investors.

**References**


**TABLE 4: COMPARISON OF AE TO MM FORECASTS**

<table>
<thead>
<tr>
<th>Month</th>
<th>AE</th>
<th>MM</th>
<th>Relative Performance of AE (Superior = 1, Inferior = 0)</th>
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<td>1</td>
<td>0.008995</td>
<td>0.007645</td>
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<tr>
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<td>1</td>
</tr>
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<td>0.001114</td>
<td>0.001820</td>
<td>1</td>
</tr>
<tr>
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<td>0.004464</td>
<td>1</td>
</tr>
<tr>
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<td>0</td>
</tr>
<tr>
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<td>0.003853</td>
<td>0.003854</td>
<td>1</td>
</tr>
<tr>
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<tr>
<td>24</td>
<td>0.004495</td>
<td>0.004631</td>
<td>1</td>
</tr>
</tbody>
</table>


Notes

1 See Campbell (2000) and Cochrane (2001) for a survey of the asset pricing literature.

2 See, for example, Black, Jensen and Scholes (1972) and Fama and MacBeth (1973). Deviations from CAPM have been reported by Banz (1981), Rosenberg, Reid and Lanstein (1985) and DeBondt and Thaler (1985), leading to the development of multifactor asset pricing models by Fama and French (1992, 1996). Australian evidence is provided by Halliwell, Heaney and Sawicki (1999) and Faff (2001).

3 The impact of randomly varying beta coefficients on Australian returns has been investigated extensively by Brooks, Faff and Lee (1992, 1994).

4 See, for example, Holden, Peel and Thompson (1985) for an overview of the literature.

5 Elliot and Anderson (1995) advocate the usefulness of memory decay in modelling an individual’s predictions of the outcome of a series, stating that “a categorisation algorithm incorporating exponential decay will adjust more quickly to changing category definitions than will an algorithm that weights past observations equally, because the exponential decay will put relatively more weight on the more recent observations. With equal weighting, adjustment to a change can occur only as the number of post-change observations becomes large relative to the number of pre-change observations”.

6 For a complete discussion, see Gujarati (1988).

7 This is an extension of the standard MM contributed by Black et al. (1972) of the form: \[ r_{it} = \alpha_i + \beta_i (R_{mt} - R_{0}) + \varepsilon_{it} \]

8 For a discussion of the impacts of infrequent trading and risk measurement, see Dimon (1979).

9 Prior to the estimation of the respective approaches of investors’ expectations, each individual return series was tested for the presence of non-stationary behaviour. Tests for stationarity were conducted using the following AR(1) scheme: \( R_t = \alpha + \phi R_{t-1} + \varepsilon_t \). Table A reports the results from this procedure. The null hypothesis in this case is \( H_0: \phi = 0 \).

10 \( R^{\text{old}} \) and \( R^{\text{new}} \) represent \( R^2 \) measures form the estimation of the MM and AE methods. Degrees of freedom in the numerator are equivalent to the number of new regressors, one in this situation. Denominator degrees of freedom are calculated by subtracting the number of parameters to be estimated in the new model from the number of data observations, 43 in this case, as two observations are lost from the original 48 after creating lags and there are three parameters to be estimated. Resultant \( F \)-statistics are tested for significance at the standard levels of tolerance.

11 Such behaviour is consistent with the characteristics of a stationary series, of which portfolio returns are an example. Hamilton (1994) shows that the effect of past observations on the current value of a stationary AR process systematically decreases with age of observation. Therefore, the effects of current information decrease in relevance in explaining returns that are further out in time. On the evidence, the importance of recent information captured by the AE approach for the Australian setting has little relevance in explaining returns that are further out in time than 12 months.

12 Importantly, Brailsford et al. (1997) note that the CAPM is a single period model and, if it is to be applied across a multi-period timeframe, a constant value of the estimate must be assumed. However, there has been much evidence to reject the notion that risk is constant through time.