How profitable is the butterfly strategy in Australian fixed income markets?

This study investigates the profitability of the butterfly strategy, which is commonly used by active bond fund managers in order to boost portfolio returns. While the strategy’s success is conditional on the accuracy of forecast shifts in the yield curve, it also has a defensive feature that may enable any losses incurred to be relatively small. We find that the cash and dollar duration-neutral butterfly does not, on average, generate positive profits after transaction costs. However, the regression-weighted butterfly often generates a statistically significant profit after transaction costs, of approximately two to three basis points per day.

Background theory

The butterfly strategy is a commonly used strategy that takes advantage of the different convexities of bonds with differing maturities. It involves buying a portfolio of bonds comprising short-maturity bonds and long-maturity bonds and simultaneously selling a portfolio of bonds comprising medium-maturity bonds. The portfolio purchased is known as a barbell portfolio, while the portfolio sold is known as a bullet portfolio. The investor has an expectation that interest rates corresponding to these maturities will then shift in a certain direction and expects to make a profit by reversing the original positions, i.e. by selling the barbell that was originally purchased and buying back the bullet that was originally sold.

Figure 1 illustrates these points. In Figure 1, we plot the relationship of bond price to interest rates for a two-year bond, a 10-year bond and a 30-year bond, which will represent short, medium and long-term bonds. Each pays 6% coupons per annum paid semi-annually and has a face value of $100.

Assuming that interest rates are initially at 6% per annum, all bonds have a price of $100. However, when interest rates drop, because the 30-year bond has a more curved price-yield relationship, its price increases more dramatically than the shorter bonds. Similarly, when interest rates rise, the 30-year bond falls in value by a larger amount than the shorter bonds because of its higher curvature or convexity relative to the shorter bonds whose price-yield relationship is more linear. Intuitively, if we purchase a barbell portfolio and interest rates drop, it is likely to increase in value by more than a bullet portfolio because of the exposure to the long bond. Less obviously, if interest rates increase, the value of the barbell portfolio will fall by less than the value of the bullet portfolio because of exposure to the short bond, and also because the long bond’s curvature in this region is not a lot higher than the curvature of the medium bond.

In this study we investigate two well-known butterfly strategies: (i) the cash and dollar duration-neutral butterfly and (ii) the regression-weighted butterfly strategy. As for all butterfly strategies there is an implicit defensive mechanism, for example, if an adverse event causes the prices of all bonds to fall then the investor has lost wealth on the barbell portfolio, but this loss can be offset by gains from the bullet portfolio.

Gains from the bullet portfolio occur when medium-maturity bonds that were initially sold are repurchased at a cheaper price.
All butterfly strategies also depend crucially on the concept of duration in determining the number of short, medium and long-term bonds that need to be traded. The duration of a bond is a first-order approximation of how bond prices change when interest rates change. In Figure 2, we consider a zero-coupon bond with a face value of $100, a term to maturity of 20 years and interest initially accruing semi-annually at 6% p.a. in nominal terms. The price of this bond is $30.66. If interest rates then increase to 7% p.a. nominal, the price of this bond becomes $25.26. Based on the concept of duration, however, the bond price is estimated to fall to $24.70. By using the concepts of duration and convexity, a closer approximation is obtained, which is effectively a second-order approximation.

Figure 2 also shows that the duration approximation works quite well when changes from initial interest rates are relatively small, but less well when interest rate changes are relatively large. Approximations based on duration and convexity work quite well, however, except for very large interest rate changes. The importance of duration in calculating the actual quantities of bonds to be traded in the cash and dollar duration-neutral and regression-weighted butterflies is discussed in more detail below.

The cash and dollar duration-neutral butterfly
To set up a cash and dollar duration-neutral butterfly, we assume that investors sell a fixed number of medium-maturity bonds, \( Q_M \) at a price of \( P_M \) each, i.e. they take a short position in a bullet portfolio. They then use these proceeds to purchase a barbell portfolio by purchasing \( Q_S \) short bonds and \( Q_L \) long bonds at a price of \( P_S \) and \( P_L \) respectively. This is reflected in Equation (1).

\[
Q_S P_S + Q_L P_L = Q_M P_M \tag{1}
\]

\[
Q_S D_S + Q_L D_L = Q_M D_M \tag{2}
\]

In addition, in setting up the cash and dollar duration-neutral butterfly we assume that the required rate of return on bonds of all maturities is forecast to change by the same amount. Thus, for example, if short bonds are being priced at a yield of 6% p.a. and the prediction is that this required rate of return will increase to 7% p.a. then the assumption is that the required rate of return on bonds of all maturities will increase by 1% or 100 basis points. This type of change in interest rates may occur if investors uniformly move money away from the bond market and into equity markets. They are thus selling bonds at all maturities, causing bond prices at all maturities to fall and yields on bonds of all maturities to rise. Similarly, if investors uniformly move money into the bond market from equity markets, for example, they may uniformly purchase bonds of all maturities, causing bond prices at all maturities to rise and bond yields at all maturities to fall. The important assumption is that changes in the yield curve are uniform across all maturities.

The strategy works by purchasing a barbell portfolio with the same duration as that of the bullet portfolio, which is initially sold. This is reflected in Equation (2) below where the terms \( D_M \), \( D_S \) and \( D_L \) represent the dollar durations of the medium, short and long bonds respectively. Equation (2) implies that if interest rates increase uniformly then, based on the concept of duration, both the barbell and the bullet will fall in value by the same amount. Thus losses from the barbell position exactly offset gains from the bullet position and no profit is expected to be made. In fact, because the barbell has a
higher convexity than the bullet, the barbell falls in value by a smaller amount than the bullet, and thus losses from the barbell are smaller than the gains from the bullet and a profit is made.

Similarly, if interest rates fall uniformly then both the barbell and the bullet increase in value. Based on the concept of duration, the gains from the barbell portfolio exactly offset the losses from the bullet position. However, because of the barbell’s greater convexity, gains from the barbell are greater than losses from the bullet and a profit is made.

The regression-weighted butterfly

The regression-weighted butterfly differs from the cash and dollar duration-neutral butterfly in two important ways. First, it is not a cash-neutral strategy meaning that the investor has to invest a positive amount of money upfront in order to generate a return. Second, the strategy can generate a profit if interest rates change in either of two ways. The first of these is the same as for the cash and dollar duration-neutral strategy, i.e. a uniform shift in interest rates across all maturities and is reflected in Equation (3).\(^6\) The second is a non-uniform shift in interest rates such that:

(i) The short rate falls by \(x\) basis points and the long rate rises by \(\beta x\) basis points and the medium rate remains unchanged; or

(ii) The short rate rises by \(\beta x\) basis points and the long rate falls by \(x\) basis points and the medium rate remains unchanged.

Equation (4) reflects that based on the concept of duration, if either scenario (i) or (ii) above occurs then this strategy is expected to return a zero profit. Looking at scenario (i), this occurs because if the short rate falls, the investor gains at the short end of the barbell portfolio but loses at the long end of the barbell, and based on the concept of duration these offset each other exactly.\(^7\) Thus the value of the barbell, is unchanged. The value of the bullet is also unchanged as there is no change in the interest rate for medium-maturity bonds.

\[
Q_S D_S + Q_L D_L = Q_M D_M \quad (3)
\]

\[
Q_S D_S = \beta Q_L D_L \quad (4)
\]

In actual fact, a profit can be achieved from scenario (i) because long bonds have a higher convexity than short bonds. As a result, any gains from the short end are actually greater than any losses at the long end and a profit is made. Also note that a profit would be made under scenario (i) without any transactions in a bullet portfolio. However, if scenario (i) does not occur and instead all bonds lose money due to a uniform upward shift in rates, then the investor would have no protection and would lose money. By selling a bullet, the investor’s losses are minimised because they can then buy back the bullet at a relatively low price to offset losses on the barbell. It should also be noted that scenario (i) can occur if investors sell long bonds and use the proceeds to buy short bonds in anticipation of an increase in interest rates. The selling of long bonds causes long bonds to drop in price and long bond yields to rise, while the purchase of short bonds causes short bonds to increase in price and their yields to fall.

Profits that occur under scenario (ii) arise for analogous reasons. Here the investor gains from a drop in rates at the long end and loses from the increase in rates at

**FIGURE 2:** Approximating changes in bond prices based on duration and convexity
the short end. However, because of the greater convexity of long bonds relative to short bonds, the gain at the long end offsets the loss at the short end and a profit is made. Scenario (ii) can occur when investors sell short bonds and use the proceeds to buy long bonds, in anticipation of a drop in interest rates.

Finally, the parameter $\beta$ is often obtained by regressing changes in interest rates at the short end on changes in interest rates at the long end and hence the name given to this form of butterfly.

**Empirical analysis**

In this section we estimate the profits from implementing the cash and dollar duration-neutral and regression-weighted butterfly strategies. We first obtain daily closing prices on active Australian Treasury bonds as of 8 September 2008. The data comes from the Datastream database. Although our sample coincides with some part of the still ongoing global financial crisis, the Reserve Bank of Australia did not take any drastic measures to cut the cash rate in this period. If the cuts had been significant, these changes in cash rate could have led to large changes in longer maturity Australian government bonds, which would have given rise to larger profits. As a result, our calculated profits should be fairly reflective of a non-volatile bond market. The bonds we use to calculate the profits from the cash and dollar duration-neutral and regression-weighted butterflies are shown in Table 1. We calculate these profits for four different combinations of long, short and medium bonds.

We calculate the profit from all portfolios on each trading day from 13 September 2007 to 8 September 2008. We also calculate the profit from portfolios 2 and 3 for each trading day from 17 January 2006 to 28 August 2007.

We calculate profits on a daily basis because the butterfly strategies are conditional on a forecast change in yields to maturity across the long, short and medium maturities. Over a given trading day, yields to maturity can be quite volatile and therefore these strategies need to be implemented over short time horizons.

For the regression-weighted butterfly, we calculate the percentage return from the strategy on day $t$ as the dollar profit adjusted for transaction costs on day $t$ divided by the net cost to set up the strategy on the preceding day.

### Results

Figure 3 shows a plot of the daily dollar profit, after transaction costs, of the cash and dollar duration-neutral strategy for portfolio one over the period 13 September 2007 to 8 September 2008. The profit is calculated for each medium-maturity bond sold. Figure 3 shows that butterfly strategies tend to give rise to relatively low losses calculated after adjusting for coupon payments in the following way:

$$\pi_{t}^{\text{Cash}} = \Delta P_{S,t} + \Delta P_{L,t} + \Delta P_{M,t} + C_{S,t} + C_{L,t} - C_{M,t}$$

(5)

$$\pi_{t}^{\text{Re.g}} = \Delta P_{S,t} + \Delta P_{L,t} + \Delta P_{M,t} + C_{S,t} + C_{L,t} - C_{M,t}$$

(5a)

The profit in dollars, after transaction costs, from the cash and dollar duration-neutral and regression-weighted butterflies are given in Equations (6) and (7) below.

$$\pi_{t}^{\text{Cash}, TC} = \left\{ \begin{array}{ll} \pi_{t}^{\text{Cash}} & \text{if } \pi_{t}^{\text{Cash}} \geq 0 \\ \pi_{t}^{\text{Cash}} & \text{if } \pi_{t}^{\text{Cash}} < 0 \end{array} \right.$$

(6)

$$\pi_{t}^{\text{Re.g}, TC} = \left\{ \begin{array}{ll} \pi_{t}^{\text{Re.g}} \times \left( 1 - \frac{\text{Cashrate}}{365} \right) & \text{if } \pi_{t}^{\text{Re.g}} \geq 0 \\ \pi_{t}^{\text{Re.g}} \times \left( 1 + \frac{\text{Cashrate}}{365} \right) & \text{if } \pi_{t}^{\text{Re.g}} < 0 \end{array} \right.$$

(7)

In Equation (6), for the cash and dollar duration-neutral butterfly, we assume that the investor faces a bid-ask spread of 5 basis points on three sets of transactions, namely buying and selling short, medium and long bonds, giving a total of 15 basis points in transaction costs.

In Equation (7), for the regression-weighted butterfly, there is an extra cost because the strategy involves a net positive cost. We assume that the investor borrows this amount for one day at the overnight cash rate. Data on the overnight cash rate is available from the website of the Reserve Bank of Australia.

We calculate profits for each medium-maturity bond sold. Figure 3 shows that butterfly strategies tend to give rise to relatively low losses.

### Table 1: Bonds used to calculate profits from the cash and dollar duration-neutral and regression-weighted butterflies

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Short</th>
<th>Barbell</th>
<th>Long</th>
<th>Medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio 1</td>
<td>7.5%: 15/9/2009</td>
<td>5.75%: 15/5/2021</td>
<td>6.25%: 15/4/2015</td>
<td></td>
</tr>
<tr>
<td>Portfolio 3</td>
<td>7.5%: 15/9/2009</td>
<td>5.25%: 15/3/2019</td>
<td>6.00%: 15/2/2017</td>
<td></td>
</tr>
<tr>
<td>Portfolio 4</td>
<td>7.5%: 15/9/2009</td>
<td>5.75%: 15/5/2021</td>
<td>5.75%: 15/6/2011</td>
<td></td>
</tr>
</tbody>
</table>

Note: For each bond the annual coupon rate and the bond’s maturity date are shown. All bonds pay coupons semi-annually.
(though there are one or two outliers), reflecting the defensive features of this strategy. This defensive feature also means that gains tend to be relatively low as well.

In Table 2, we show the distribution of dollar profits for the four portfolios in our analysis. It shows that while profits are achieved on roughly 50% of days in our sample, after transaction costs, no portfolio returns a profit on average. In each case, the average loss is less than one cent for each medium-maturity bond sold. However, for the period 17 January 2006 to 28 August 2007, portfolios 2 and 3 both return a very small average profit, after transaction costs, of 0.06 cents and 0.05 cents for each medium-maturity bond sold. We also constructed 95% confidence intervals for the mean profit for all portfolios and found that none were statistically different from zero.

Table 2 shows that the profitability of the strategy depends on when it is implemented. It also shows that bond fund managers should choose carefully the days on which they implement the strategy because there are days on which significant profits can be made. For example, for portfolio one, on 5 August 2008, this strategy earns 62.85 cents for each medium-maturity bond sold or $62.85 for every 100 medium-maturity bonds sold. The finding that it is difficult to profit from the cash and dollar duration-neutral strategy should not be surprising as this would imply that a net zero-cost strategy can consistently realise profits. Intuitively, this seems unlikely, though such a finding is not inconsistent with the efficient markets hypothesis as this is not a risk-free zero-cost strategy. For this strategy, the major risk is that the yield curve does not shift in a parallel fashion. Other risks may include observed bond prices being attributable to liquidity trades and hence not consistent with expected shifts in the yield curve.

In Table 3, we provide statistics on the daily percentage profits from implementing the regression-weighted butterfly strategy. In our calculations, we use four values of the beta parameter, namely $\beta = 0.25$, $\beta = 0.50$, $\beta = 0.75$ and $\beta = 1.00$. The beta parameter can also be estimated from a regression of changes in the yields to long bonds on changes in yields to short bonds.
Table 3 shows that, unlike the cash and dollar duration-neutral strategy, the regression-weighted strategy does realise a positive profit on average and this is often statistically significant. The proportion of days on which the strategy returns a profit is also higher relative to the cash and neutral butterfly. This reflects the fact that the regression-weighted butterfly is designed to return a profit when the yield curve shifts in a parallel fashion or when it steepens or flattens, whereas the cash and dollar duration-neutral strategy is designed to return a profit only from a parallel shift in the yield curve.

Table 3 also compares the daily returns from the regression-weighted butterfly strategy with the daily returns to an index of Australian government bonds designed to have a constant duration of seven years. This index is a good approximation of the index that might be held by a passive bond manager. In doing this comparison, we also take into account the volatility of daily returns and further note that a passive index manager is more concerned with long-term performance and less concerned with daily fluctuations compared with an active manager.

From Table 3, portfolio 1 and portfolio 3 provide roughly the same average daily return as the index, 2.6–2.7 basis points per day compared with 2.9 basis points for the index, but a much lower volatility and therefore a statistically significant and positive profit, whereas the index average daily return is statistically not different from zero. Portfolio 2 gives a statistically significant and positive return for $\beta = 1$ while the index does not provide a daily average return that is statistically greater than zero. Portfolio 3 provides statistically significant and positive returns for all values of beta. Only portfolio 4 does not provide any beta values that give rise to a statistically significant and positive return, but this is true of the index as well.

### TABLE 3: Daily percentage profits from regression-weighted butterfly after transaction costs for different values of $\beta$ for the period 13/09/2007 to 8/9/2008

**Panel A: Profits from portfolio 1**

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Std Dev</th>
<th>95% CI for mean</th>
<th>Profitable days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-1.26%</td>
<td>1.14%</td>
<td>0.027%</td>
<td>0.188%</td>
<td>(0.00%, 0.05%)</td>
<td>61.7%</td>
</tr>
<tr>
<td>0.50</td>
<td>-0.45%</td>
<td>0.52%</td>
<td>0.026%</td>
<td>0.085%</td>
<td>(0.02%, 0.04%)</td>
<td>67.5%</td>
</tr>
<tr>
<td>0.75</td>
<td>-0.26%</td>
<td>0.37%</td>
<td>0.026%</td>
<td>0.070%</td>
<td>(0.02%, 0.04%)</td>
<td>70.4%</td>
</tr>
<tr>
<td>1.00</td>
<td>-0.23%</td>
<td>0.30%</td>
<td>0.026%</td>
<td>0.066%</td>
<td>(0.02%, 0.03%)</td>
<td>70.0%</td>
</tr>
<tr>
<td>INDEX</td>
<td>-0.86%</td>
<td>1.29%</td>
<td>0.029%</td>
<td>0.386%</td>
<td>(-0.02%, 0.08%)</td>
<td>54.1%</td>
</tr>
</tbody>
</table>

**Panel B: Profits from portfolio 2**

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Std Dev</th>
<th>95% CI for mean</th>
<th>Profitable days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.50</td>
<td>-1.94%</td>
<td>2.23%</td>
<td>0.036%</td>
<td>0.482%</td>
<td>(-0.03%, 0.10%)</td>
<td>53.3%</td>
</tr>
<tr>
<td>0.75</td>
<td>-1.31%</td>
<td>1.51%</td>
<td>0.032%</td>
<td>0.297%</td>
<td>(-0.01%, 0.07%)</td>
<td>57.5%</td>
</tr>
<tr>
<td>1.00</td>
<td>-1.13%</td>
<td>1.31%</td>
<td>0.032%</td>
<td>0.255%</td>
<td>(0.00%, 0.06%)</td>
<td>56.3%</td>
</tr>
<tr>
<td>INDEX</td>
<td>-0.86%</td>
<td>1.29%</td>
<td>0.029%</td>
<td>0.386%</td>
<td>(-0.02%, 0.08%)</td>
<td>54.1%</td>
</tr>
</tbody>
</table>

**Panel C: Profits from portfolio 3**

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Std Dev</th>
<th>95% CI for mean</th>
<th>Profitable days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-0.30%</td>
<td>0.31%</td>
<td>0.026%</td>
<td>0.078%</td>
<td>(0.02%, 0.04%)</td>
<td>67.9%</td>
</tr>
<tr>
<td>0.50</td>
<td>-0.27%</td>
<td>0.21%</td>
<td>0.026%</td>
<td>0.068%</td>
<td>(0.02%, 0.03%)</td>
<td>68.8%</td>
</tr>
<tr>
<td>0.75</td>
<td>-0.27%</td>
<td>0.18%</td>
<td>0.026%</td>
<td>0.066%</td>
<td>(0.02%, 0.03%)</td>
<td>67.5%</td>
</tr>
<tr>
<td>1.00</td>
<td>-0.27%</td>
<td>0.18%</td>
<td>0.026%</td>
<td>0.066%</td>
<td>(0.02%, 0.03%)</td>
<td>67.5%</td>
</tr>
<tr>
<td>INDEX</td>
<td>-0.86%</td>
<td>1.29%</td>
<td>0.029%</td>
<td>0.386%</td>
<td>(-0.02%, 0.08%)</td>
<td>54.1%</td>
</tr>
</tbody>
</table>

**Panel D: Profits from portfolio 4**

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>Std Dev</th>
<th>95% CI for mean</th>
<th>Profitable days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.50</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.75</td>
<td>-3.15%</td>
<td>3.36%</td>
<td>0.021%</td>
<td>0.983%</td>
<td>(-0.12%, 0.16%)</td>
<td>51.9%</td>
</tr>
<tr>
<td>1.00</td>
<td>-1.51%</td>
<td>1.18%</td>
<td>0.021%</td>
<td>0.363%</td>
<td>(-0.02%, 0.07%)</td>
<td>53.8%</td>
</tr>
<tr>
<td>INDEX</td>
<td>-0.86%</td>
<td>1.29%</td>
<td>0.029%</td>
<td>0.386%</td>
<td>(-0.02%, 0.08%)</td>
<td>54.1%</td>
</tr>
</tbody>
</table>
In making these comparisons with the index, however, it should be noted that an index manager is not concerned by daily fluctuations in the same way that an investor in the butterfly strategy would be. Passive managers most likely have significantly longer investment horizons of at least one year when their performance is evaluated. Active managers, on the other hand, are constantly looking for profitable trading opportunities so this comparison may be flawed.

Conclusion
In this study, we estimated the profitability of two types of butterfly strategies. Butterfly strategies are commonly used by active fixed income managers to enhance portfolio returns. We found that after transaction costs it is very difficult to profit from the cash and dollar duration-neutral butterfly strategy, though the strategy does return profits on approximately 50% of trading days. The average profit from this strategy is not statistically different from zero. We found that the regression-weighted butterfly can generate profits on up to 70% of trading days and seems to yield profits, after transaction costs, of roughly two to three basis points per day. These positive profits are often statistically significant. On a daily basis, the regression-weighted butterfly seems to provide better risk-adjusted returns relative to a bond index that might be held by a passive investor, though this comparison seems flawed given the different investment horizons of passive and active managers.

This study is important as a growing number of investors will invest funds in the bond market instead of equity markets, in light of the current global financial crisis. These investors may invest some wealth with active bond fund managers, and this study provides insight into the profitability of a strategy commonly used by active bond fund managers.

There are many areas for further research including: the role of transaction costs in implementation of the butterfly strategy; what the appropriate benchmark should be in evaluating the profitability of the butterfly strategy; and the effect of different holding periods on the strategy's profitability.

Notes
1 The author gratefully acknowledges the research assistance of Chong Qian, Adrian Bigaj, Nixuan Wang, Hugh Denton and Wanye Song and the comments of an anonymous referee.
2 Some practitioners refer to four types of butterfly strategies: (i) the cash and dollar duration-neutral butterfly; (ii) the regression-weighted butterfly; (iii) the 50–50 butterfly and (iv) the maturity-weighted butterfly. However, it can be shown that the 50–50 and maturity-weighted butterfly strategies are special cases of the regression-weighted butterfly. See, for example, Grieves (1999), Martellini et al. (2002) and Martellini et al. (2006).
3 Note the bond price is very low because there are no coupons and because of the very long term to maturity.
4 The dollar duration of a bond is the amount in cents that a bond's price falls (rises) if interest rates corresponding to that bond's maturity increase (decrease) by 100 basis points. Dollar durations of longer bonds are generally higher than those of shorter bonds depending on the coupon rates of the two bonds.
5 Although the short bonds will fall in price by a lower amount than the long bonds and the medium bonds, the quantities of short, medium and long bonds traded ensure that the losses, based on duration, are exactly offsetting.
6 Equation (3) is identical to Equation (2) from the cash and dollar duration-neutral strategy.
7 Again, although the short bonds will fall in price by a lower amount than the long bonds and the medium bonds, the quantities of short, medium and long bonds traded ensure that the losses, based on duration, are exactly offsetting.
8 See Martellini et al. (2006), p. 242 for an example of how profits increase from larger shifts in the yield curve.
9 $C_{50}$ is the drop in bond prices on day $t$ for the short bond assuming that $t$ is an ex-coupon date for the short bond. $C_{50}$ and $C_w$ are defined similarly for the long and medium bonds. This adjustment is necessary because, for example, buying a bond today for $100 and then selling it immediately after the bond becomes ex-coupon for $97 does not represent a loss if the coupon to be paid is three dollars. Instead of a $100 bond, you now have a $97 bond and $3 in cash, representing the coupon payment.
10 This may underestimate the cost of funding for individual investors or non-bank institutions. However, if the positions are held for one day, the interest cost (being equal to accrued interest for one day) should be negligible, even for more expensive funding options. If the positions are reversed over one month, for example, the one-month Bank Bill Swap Rate might be a good proxy for the interest cost.
11 In our calculations, we exclude days in our sample where the net cost is negative as this is inconsistent with the underlying theory, which says that investors must pay for a net positive convexity.
12 See, for example, Martellini et al. (2006), p. 218 where the index in their example has a duration of 6.73 years.
13 No statistics are reported for portfolios 2 and 4 for $\beta = 0.25$ as all days in the sample then give rise to a negative net cost. No statistics are reported for portfolio 4 for $\beta = 0.50$ as there are only 31 daily observations out of a possible 240 observations since all other days in the sample give rise to a negative net cost.

References