The importance of sustainable spending rates by retirees has been underscored by rapid population ageing and the lacklustre performance of markets and pension funds in the post-GFC period. This suggests that financial planners and advisors should pay more attention to the estimation of risk in retirement finance modelling in their analyses and advice to clients. This paper provides some useful guidance on the application of two available techniques in this regard.

There are several contexts in which it is important to determine the optimal rate of withdrawal from an investment portfolio while ensuring the withdrawal is sustainable over a desired horizon. The most common is the situation faced by a new retiree whose life savings are held in an investment portfolio. The retiree has to make monthly withdrawals from the savings pool for living expenses, while ensuring that the portfolio value is maintained at a level sufficient to support the withdrawal stream for the rest of their life. If the withdrawal rate is too high the retiree faces the prospect of running down the savings pool prematurely, resulting in financial ruin. If the withdrawal rate is too low the retiree is deprived of a better standard of living during their twilight years.

How does the retiree determine the optimal amount to withdraw while maintaining some minimum safety level in the portfolio value if investment returns and remaining life span are uncertain? Regardless of whether the retiree wishes to target a portfolio value of zero at the end of life or some positive value to leave as a bequest (or as an additional safety measure) the same computational issues apply.

These computational issues also arise in other contexts, for example, the situation faced by the trustees of an endowment, such as a scholarship fund. How do the trustees determine the maximum sustainable scholarships that can be offered while maintaining desired safety levels in portfolio value? Financial institutions such as insurance companies face a similar question in pricing annuity products. What risk of ruin is involved in promising a fixed payment stream to the annuitant given projected, but uncertain, investment returns on the sum received over the life of the product?

The issue of sustainable spending rates for retirees has taken on an added dimension since the global financial crisis (GFC) due to the substantial losses that pension funds suffered during the 2007-08 period, and the lacklustre performance of markets and funds in the post-GFC period. The importance of sustainable spending rates is also underscored by the fact that the Australian population is ageing rapidly, and is expected to have an increased life expectancy. The Treasury projects that in 2050 nearly 25 per cent of the population will be aged 65 and over, compared with 13 per cent today.1 Longer post-retirement life spans combined with the possibility of lower retirement savings balances gives increased importance to this issue of sustainable withdrawals. Some commentators suggest that the financial planning industry pays too little attention to the concept of “sustainable spending”, which is key to effective strategic planning for corporate pensions, public pensions, foundations, and endowments — even for individuals’.

Despite the uncertainties involved in predicting future financial returns, perhaps a sensible approach to retirement financial planning would be to base expectations on the recent past experiences and performance of markets, with some added sensitivity analyses for estimating risk. The financial experiment conducted in this paper follows this approach, with future projections being made by replicating the recent past market performances in a bootstrap simulation framework, for the purposes of formulating sustainable withdrawal rates and estimating risks.
associated with those outcomes as measured by the probabilities of financial ruin. This approach is compared with a simulation-free analytical approach proposed in Milevsky and Robinson (2005).

**Related literature**

A computational approach to sustainable withdrawal rate calculations

The situation faced by a new retiree planning their future retirement finances is considered. At the time of retirement their life savings are assumed to be held in an investment portfolio with fixed monthly withdrawals made from the portfolio for living expenses. The retiree’s remaining life span and the returns from the investment portfolio are uncertain. The retiree’s objective is to determine the fixed spending rate that is sustainable over the rest of their life with some minimum level of confidence.

Put another way, the retiree is seeking to determine the probability of facing financial ruin for a given spending rate. Milevsky and Robinson (2005) illustrate how the spending rate, longevity and investment returns are linked with the probability of financial ruin, by means of the retirement finances triangle shown below. Longevity, investment returns and the consumption rate are shown at the nodes of the triangle, each affecting the probability of financial ruin, represented by the mass inside the triangle. An increase in the spending rate, increase in longevity or a reduction of the investment returns will increase the probability of financial ruin.2

![FIGURE 1: The retirement finances triangle — based on Milevsky and Robinson (2005)](image)

The retiree outlives their resources and faces financial ruin if the present value of the future spending stream exceeds the initial investment pool, $w$. In the simplistic case where the investment returns and longevity are known with certainty, and if $T$ is the time of death, and $R$ is the investment return each period, the present value of a future spending stream of $1$ each period can be expressed as $PV$, the present value of an annuity, given as:

$$PV = \sum_{i=1}^{T} \frac{1}{(1 + R)^i}$$

If $c$ is the number of dollars the retiree plans to spend each year, financial ruin results if $c.PV > w$. Since $PV$, in this case, depends only on $R$ and $T$, which are known, whether $c$ and $w$ are consistent is easily established.

When the life span of the retiree and the investment returns are both uncertain, the present value of a spending stream of $1$, referred to as the Stochastic Present Value ($SPV$) takes the following form:

$$SPV = \frac{1}{(1 + \bar{R})} + \frac{1}{(1 + \bar{R})(1 + \bar{R})} + \ldots + \frac{1}{(1 + \bar{R})^T}$$

$$= \sum_{i=1}^{T} \Pi_{i-1} (1 + \bar{R})^i$$

Here $\bar{T}$ is the random time of death, $\bar{R}$ is the random investment return and $j$ is any period within the life span $T$. The probability of financial ruin is then equal to the probability that the $SPV$ (multiplied by the planned annual consumption) is greater than the initial savings pool. To compute the probability of retirement ruin the value of the $SPV$ has to be computed.

The value of the $SPV$ is not readily found analytically in this stochastic case. However, Milevsky and Robinson (2005) provide an analytical approach to solving for the $SPV$ by making some simplifying assumptions regarding the probability distributions of the stochastic variables. They derive a closed form solution under the assumption that returns are lognormally distributed and lifetimes are exponentially distributed. They show that the probability of $SPV$ (for a planned consumption stream of $1$) being greater than the value of the retirement savings pool $w$ is given by the Gamma distribution evaluated at the parameters of the distribution, and is shown as follows:3

$$\text{Prob}(SPV > w) = \text{GammaDist} \left[ \frac{(2\mu + 4\lambda)}{(\sigma^2 + \mu)}, 1, \frac{\sigma^2 + \lambda}{2}, \frac{1}{w} \right]$$

where, $\mu$ and $\sigma$ are the return and volatility of the investment portfolio, and $\lambda$ is the mortality rate. For given values of these parameters, the probability of $SPV$ being larger than $w$, which is the probability of financial ruin, can be computed on an Excel spreadsheet or with a statistical computer package.
**Data-centric approaches**
Using more data-centric approaches, researchers in the financial planning industry have been investigating the issue of estimating a maximum sustainable withdrawal rate that will allow a retirement asset pool to fund a full retirement period. An early study that has significantly influenced industry practice is Bengen (1994). In this study, 65 years of US equity and bond market data between 1926 and 1991 were analysed to search for the highest withdrawal rate that would sustain an equally weighted portfolio of stocks and bonds over a 30-year retirement horizon. Based on his results, Bengen prescribes a 4 per cent withdrawal rate. This rule has come to be known as the ‘4 per cent rule’ and investment advisors in the United States, Australia and other developed markets widely follow the policy of recommending an initial withdrawal rate of 4 per cent to 6 per cent for retirement portfolios. The nominal withdrawal amount is increased each year by the rate of inflation over a retirement horizon, usually assumed as 30 years.

Studies that followed Bengen (1994) have examined many variations on this basic theme. Pye (2000) and others have utilised the bootstrap simulation because it provides a simple framework for mimicking investors who make sustainable withdrawals, and for evaluating investment choices over time without needing to make any further assumptions about risk preferences. Basu, Byrne and Drew (2011) take this approach to examine the accumulation phase of retirement planning, and the benefits or otherwise of asset allocation changes from a pure glidepath that is commonly adopted by life cycle or target date retirement funds. The focus of many of these studies has been on investors and markets based in the United States but the issue of sustainability in retirement planning has received limited attention in Australia.

In this paper, we apply a bootstrap simulation approach to investigate ruin probabilities in retirement portfolios. The methodology of Milevsky and Robinson (2005) is also applied to the Australian context as an alternative approach. The results of this approach form a useful basis for comparing the results of the bootstrap approach.

**Data**
The data used in the study were sourced from the Thomson Reuters Datastream database. The Datastream-computed benchmark stock market index represents the Australian stock market, and the proxy for the bond market is the Citibank All Maturities Government Bond Index. The data for the bond index commences from December 1984. The stock index commences in 1973 but we select December 1984 as the common start date for both indexes. The sample period ends at February 2012. An advantage in using a sample period that covers the more recent past rather than a long history is that the data is more representative of the recent market experience. This period covers a mix of bull and bear market episodes.

**Bootstrap methodology and the simulation approach**
Initially, the bootstrap re-sampling technique was used in the field of statistics to examine the probability distribution of sampling outcomes, but it is now widely used in finance for simulation purposes. The historical experience is assumed to reflect some unknown probability distribution, and future projections are derived by randomly re-sampling the data with replacement. This enables the generation of a sufficient number of independent samples needed to generate long series of return paths (which may be longer than the historical data available) and a distribution of end-of-period wealth outcomes. Because the re-sampling is done with replacement, a particular data point from the original dataset can appear multiple times in a given bootstrap sample. The probability distribution of future outcomes reflects the characteristics of the past data.

To compute bootstrap samples, we start with the set of monthly continuously compounded returns observations in the data series and repeatedly draw random observations with replacement until the required number of observations is drawn for a single complete run of wealth path computations. This results in one bootstrap sample. By repeating this process 500 times, we obtain 500 sample runs with randomly selected starting observations for the investment returns. The computational results from the 500 samples provide a frequency distribution of values from which probabilities of outcomes can be computed.

For illustrative purposes, we assume the retiree is 60 years of age at the beginning of their retirement with a savings pool of $100, and plans for a 30-year retirement period, during which 360 monthly withdrawals are made. Wealth paths are simulated for withdrawal rates of 4 per cent, 5 per cent and 6 per cent p.a. on the initial wealth. The investment returns are stated in nominal terms, which incorporate inflation effects. To convert the withdrawals to nominal terms, we raise the monthly withdrawal amount by the monthly inflation rate, based on an annual inflation rate of 3 per cent, the estimate for the future inflation rate in Australia.

We examine three investment scenarios, first an all-equity portfolio wholly invested in the stock index, second an equal-weighted portfolio invested in stocks and bonds, and third a portfolio invested initially in stocks and bonds with equal weights, but with weights progressively shifting entirely to bonds at the end of the 30-year period. This would represent a glidepath that is popular in the funds management
industry for more conservative investors. The probability of survival is defined as the frequency of ending up with a non-negative level of wealth at the end of the retirement horizon in the selected investment strategy. Conversely, the probability of financial ruin is one minus the probability of survival.

The model is calibrated as follows. The value of the investment portfolio at the end of month \( t \) is \( V_t \). The return of the portfolio for the month is \( R_t \) consisting of the capital gains and other cash returns such as dividends, assuming monthly compounding. The withdrawal \( W_t \) is assumed to be made at the end of each month. The value of the investment portfolio in the following period \( t+1 \) is then:

\[
V_{t+1} = V_t (1 + R_t) - W_t
\]

The value of an investment portfolio consisting of stocks and bonds invested in the proportion \( \lambda \) in stocks and \( 1 - \lambda \) in bonds is:

\[
V_{t+1} = V_t (1 + \lambda R_{t, \text{stock}} + (1 - \lambda R_{t, \text{bond}})) - W_t
\]

The withdrawal amount is increased at the rate of monthly inflation 0.0025, so as to maintain a constant level of real consumption.

We compute the probabilities of financial ruin based on the distributional return and risk properties of the Australian stock and bond indexes, and maintain the same assumptions of an initial savings pool of $100 and a retirement period of 30 years. The results are computed for the all-equity strategy, a balanced equity/bond strategy and for the glidepath strategy. 

**Results**

The summary statistics of the sample data are reported in Table 1. Note, for later comparison with the analytical approach, negative skewness is observed in the equity index returns.

Table 2 below shows the probabilities of financial ruin at the withdrawal rates of 4 per cent, 5 per cent and 6 per cent for the three investment strategies. As expected, for any given investment strategy, the ruin probability increases as the withdrawal rate is increased. Lower risk portfolios (inclusion of bonds or the glidepath approach) lower the risk of ruin for any given withdrawal rate.

One feature of these simulations is that the distributions of end-of-life wealth are positively skewed and have quite large mean values (relative to the initial wealth). The expected positive bequests implied reflect the fact that the average annual rates of return in the historical sample (11.3 per cent and 8.2 per cent p.a. for equities and bonds, respectively) are significantly above the chosen initial withdrawal rates (4 per cent, 5 per cent, and 6 per cent). Thus, in general, withdrawals are below the return on the portfolio such that capital tends to increase. Outcomes of large bequests and low likelihoods of financial ruin observed with, for example, the 4 per cent withdrawal rate may well be regarded by some retirees as too conservative.

Table 3 shows the probabilities of financial ruin derived from the solution to the SPV values based on equation (1) in the Milevsky and Robinson (2005) approach. In this computation the return and standard deviation assumed for the equity portfolio correspond to the average annual return and standard deviation of the equity index over the sample period, and the return and standard deviation of the balanced equity/bond strategy correspond to the portfolio return and standard deviation of the 50/50 equity and bond indexes. The mortality rate \( \lambda \) corresponds to the median remaining life time at the given age, which we assume as 30 years at the retirement age of 60. The 4 per cent, 5 per cent and 6 per cent withdrawal rates are maintained for this approach as well. The glidepath strategy that was applied to the bootstrap approach cannot be applied to this approach as it cannot accommodate sequential changes in the portfolio composition in the time path.

The ruin probabilities calculated from the analytical approach are observed to be lower than the values computed with the bootstrap approach for the corresponding investment strategies. This may be due to the fact that the analytical approach assumes a lognormal distribution for the returns, which may not be a valid assumption for the realised return distributions used in the bootstrap, which shows negative skewness and high kurtosis.

---

**TABLE 1: Summary statistics of sample data — based on monthly continuously compounded returns**

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean return</th>
<th>Median</th>
<th>Standard deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity index</td>
<td>0.00939</td>
<td>0.01298</td>
<td>0.0500</td>
<td>-2.41</td>
<td>20.15</td>
</tr>
<tr>
<td>Bond index</td>
<td>0.00684</td>
<td>0.0082</td>
<td>0.07</td>
<td>1.48</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2: Probabilities of financial ruin — based on the bootstrap simulation approach**

<table>
<thead>
<tr>
<th>Annual withdrawal rate</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>All equity</td>
<td>0.032</td>
<td>0.084</td>
<td>0.124</td>
</tr>
<tr>
<td>Equity plus bonds</td>
<td>0.019</td>
<td>0.043</td>
<td>0.080</td>
</tr>
<tr>
<td>Glidepath</td>
<td>0.000</td>
<td>0.011</td>
<td>0.030</td>
</tr>
</tbody>
</table>

*The portfolio has an annual return of 0.0974% and a standard deviation of 0.0826.

**TABLE 3: Probabilities of financial ruin — based on the analytical approach of Milevsky and Robinson (2005)**

<table>
<thead>
<tr>
<th>Annual withdrawal rate</th>
<th>4%</th>
<th>5%</th>
<th>6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>All equity</td>
<td>0.019</td>
<td>0.043</td>
<td>0.080</td>
</tr>
<tr>
<td>Equity plus bonds*</td>
<td>0.000</td>
<td>0.011</td>
<td>0.030</td>
</tr>
</tbody>
</table>

*The portfolio has an annual return of 0.0974% and a standard deviation of 0.0826.
The 4 per cent withdrawal rate was observed to result in quite low ruin probabilities with both the analytical and simulation approaches. As such, the ‘4 per cent rule’ may well be regarded by even risk-averse retirees to be a rather conservative rule. A comparison of the results from the two approaches shows that the ruin probabilities of the bootstrapping approach are higher in comparison with the ruin probabilities from the analytical approach.

Conclusions and future research directions

In this paper we have demonstrated the application of two different approaches to the estimation of risk in retirement finance in the Australian context; the bootstrap simulation approach, and the analytical approach of Milevsky and Robinson (2005) that has been made application-friendly with some simplifying assumptions. Financial planners and advisors should pay more attention to the estimation of risk in retirement finance modelling in their analyses and in their advice to clients. This paper provides some useful guidance on the application of two available techniques in this regard.

The 4 per cent withdrawal rate was observed to result in quite low ruin probabilities with both the analytical and simulation approaches. As such, the ‘4 per cent rule’ may well be regarded by even risk-averse retirees to be a rather conservative rule. A comparison of the results from the two approaches shows that the ruin probabilities of the bootstrapping approach are higher in comparison with the ruin probabilities from the analytical approach. This may be due to the fact that the bootstrap simulation would build in the negative skewness observed in the actual return distributions while the analytical approach works on the assumption of a positively skewed lognormal distribution. In that respect, the more recent market experience is better represented in the simulation approach. Whether this experience will persist in the future is, however, a matter of conjecture. Retirement planners using these techniques should be cognisant of these assumptions.

This study has not taken into consideration taxes and transaction costs such as investment expenses and their effects on the sustainable withdrawal rates. Extensions of this analysis can include these factors, and also consider the inclusion of other asset classes and further investment/withdrawal strategies.

Notes

2. Financial ruin refers to the exhaustion of the retirement savings pool. This study does not consider other assets and other income streams available to the retiree, including the age pension, to which Australian retirees have recourse in the event of financial hardship.
5. The stock indices computed here make no adjustments for the value of imputation tax credits which came into effect after July 1987. While there are several schools of thought regarding the value of franking credits, making no adjustments for franking credits corresponds to the view that franking credits carry no value.
6. While there are several Australian bond indices available in Datastream the index chosen for the study goes back the furthest in time.
7. A detailed description of the bootstrap resampling technique can be found in Efron (1979).
8. The withdrawal rates of 4 per cent, 5 per cent and 6 per cent correspond to the minimum withdrawal rates recommended by the Australian Taxation Office for allocated pensions for age groups 55–64, 65–74 and 75–79. The rates were reduced to 3 per cent, 3.75 per cent and 4.5 per cent for the 2011 and 2012 tax years.
9. Retirement Planner Calculators such as those available on the Moneysmart website of ASIC (https://www.moneysmart.gov.au/tools-and-resources/calculators-and-tools/retirement-planner) provide projections of retirement income streams and future wealth balances, for a given set of parameter inputs. The projections are based on assumed average future investment returns, not on simulations of actual market data. Retirement calculators are not designed to provide probability risk measures of outcomes.
10. The annualised equity risk premium over the sample period is 3.1 per cent, which is lower than the risk premium of 6.3 per cent reported by Brailsford, Handley and Maheswaran (2012) over a century of return data. This may be due to the influence of the high bond returns recorded over the downward trend in interest rates during the disinflation period of the 1990s.
11. For example, end-of-life wealth is $1,944, $1,178 and $977 in the all-equity, equity plus bonds and glidepath strategy, respectively.

References