EVERY ASPECT OF A RISKY BUSINESS affects its cost of capital

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This paper reveals some surprising implications of the capital asset pricing model (CAPM) which accord with common sense but not necessarily with finance textbooks. The key finding is that the cost of capital or discount rate applying to a given firm or business venture is affected by every aspect of that business, not just by its market risks or uncertainties. For example, if the firm changes its CEO, it will invariably attract a new cost of capital, either higher or lower than previously. Although this finding might seem to contradict finance theory, it can be traced back to a long-forgotten paper on CAPM by Eugene Fama.

Every aspect of a firm’s operations, or of an individual investment project (e.g. a mine, factory or hotel) affects its forward-looking cost of capital or discount rate. Contrary to conventional wisdom, there is no such thing as ‘unpriced’ project risk. Although this might not sound correct, it follows immediately from the conventional capital asset pricing model and, perhaps surprisingly, was demonstrated by one of the great founders of finance theory, Eugene Fama.

The purpose of this paper is to explain this claim and its economic intuition. I also describe some interesting implications for corporate finance thinking and practice.

The proof
In simple terms, the discount rate applicable to a project or business depends on both its ex ante (forward-looking) mean or expected net cash payoff and its similarly subjective ex ante payoff covariance (i.e. covariance with the ‘market’ or sum of all assets in the market). Hence, any characteristic of a project or firm (e.g. scale of production, operating leverage, quality of management and governance) that alters market perceptions of either of these two subjective parameters must also affect its CAPM discount rate. In other words, the market imposes a discount rate driven by its view of both the mean and covariance parameters of the future cash flow.

Consider a simplified business venture that produces a random (i.e. risky) period-end cash payoff labelled $V$, and let the current market price (i.e. the ex ante CAPM valuation) of that payoff be represented by $P$. Now write the CAPM in its ‘certainty equivalent’ or payoffs form, as set out in well-known textbooks like Brealey et al. (2014). That is

$$P = \frac{E[V] - k \text{cov}(V, r_M)}{1 + r_f},$$

where $r_f$ is the market return, $r_f$ is the risk-free rate and $k$ is a positive constant that captures the level of risk aversion in the market (larger $k$ shows greater risk aversion).

The discount rate or expected return on the venture is therefore

$$E[r] = \frac{E[V]}{P} - 1.$$
Substituting for \( P \) from the first equation gives

\[
E[R] = \frac{E[V]}{E[V] - k \text{cov}(V, r_M)} = R_f \left[ 1 - k \left( \frac{\text{cov}(V, r_M)}{E[V]} \right) \right]^{-1}
\]

where, for convenience in notation, \( R = (1 + r) \) and \( R_f = (1 + r_f) \). Hence, the CAPM discount rate applicable to a project or firm depends on the simple ratio of its payoff covariance to payoff mean

\[
\frac{\text{cov}(V, r_M)}{E[V]}
\]

I call this ratio ‘Fama’s ratio’ because it was first noted by Eugene Fama (1977). It was rediscovered by Lambert et al. (2007) and has also been given attention by Gao (2010), Christensen et al. (2010), Core et al. (2014), Johnstone (2015a, 2015b, 2016), Bertomeu and Cheynel (2015), Paugam and Ramond (2015) and Johnstone and Wagenhofer (2015).

There are several other ways to show the same point using simple equations derived easily from the CAPM. First, the conventional forward-looking returns beta of a project, that is \( \beta = \frac{\text{cov}(r, r_M)}{\text{var}(r_M)} \), can be re-written algebraically as

\[
\beta = c \left( \frac{E[V]}{\text{cov}(V, r_M)} \right) - c'\left[ 1 - k \left( \frac{\text{cov}(V, r_M)}{E[V]} \right) \right]^{-1}
\]

where \( c \) and \( c' \) are market-level constants that are immaterially affected by the properties of one relatively small business project or firm within the whole market set of firms or assets. See Johnstone (2016) for derivation. This equation reveals how the conventional CAPM ‘beta’ is affected by the project’s underlying attributes or fundamentals, and specifically by Fama’s ratio. Note again that in principle beta is a forward-looking estimate, and is essentially a summary of the more underlying estimates, namely the estimated mean payoff from the project and the estimated covariance of that payoff with the market.

It is commonly mentioned in corporate finance theory that a project’s operating leverage affects its beta. The equation above shows a mechanism by which this happens. Put simply, if we change a project’s design (e.g. the factory set-up) we change its physical and statistical nature and hence also its CAPM cost of capital. For example, suppose we install new robotic machinery at the expense of workers. This will change the make-up of costs and the volume and efficiency of production, and must inevitably alter both the estimated mean cash payoff and the estimated covariance of that payoff with the market.

It is important to note at this point that the cost of capital is not the firm’s primary concern. Primarily, the firm attempts to maximise the CAPM value of the asset, \( P \), which is determined by a trade-off between its payoff mean and covariance. In principle, the firm will settle on a production set-up that maximises \( P = E[V] / E[R] \), where \( E[R] \) itself is affected by the ratio of \( E[V] \) to \( \text{cov}(V, r_M) \). This sounds circular, because that is exactly how the CAPM equilibrium mechanism works. A way of simplifying this is to say that when making decisions the firm imagines different production set-ups, each with a different forward-looking payoff mean and covariance, and searches for the parameter pairing \( \{E[V], \text{cov}(V, r_M)\} \) that gives maximum \( P \). This is the pair with the best ratio of numerator \( E[V] \) to denominator \( E[R] \) where, interestingly, \( E[V] \) affects both the numerator (obviously) and the denominator (much less obviously).

Another way to demonstrate the role of Fama’s ratio is to write the certainty equivalent expression of the CAPM in a very different way than usual. In textbooks, the certainty equivalent \( CE \) of random payoff \( V \) is written as its mean minus an absolute dollar-amount penalty for its payoff covariance, in the same way as occurs in the numerator of the first equation in this paper. Now, rather than subtracting an additive penalty for risk, it is equally valid to write an equivalent multiplicative penalty. We can define this penalty factor \( f \) simply by writing

\[
CE = E[V] - k \text{cov}(V, r_M) = E[V] \times f.
\]
Hence, the multiplicative way to write the penalty for risk is

\[ f = \left( 1 - k \frac{\text{cov}(V, r_M)}{E[V]} \right) . \]

The CAPM value \( P \) of a risky payoff is then its certainty equivalent \( CE \), given by \( f \times E[V] \), discounted as usual at the risk-free rate. This is a very elegant way to write the CAPM, and reveals neatly how Fama's ratio drives the penalty for risk.¹

Note that \( \text{cov}(V, r_M) = 0 \) implies \( f = 1 \), meaning that only risky projects are discounted for risk, of course. However, when \( \text{cov}(V, r_M) \neq 0 \), the multiplicative discount factor \( f \neq 1 \) is influenced not only by \( \text{cov}(V, r_M) \) but by its amount relative to the mean payoff \( E[V] \). This reveals the mistake in common thinking. Specifically, the fact that only risky cash payoffs are discounted does not imply that the risk-adjusted discount factor is affected only by payoff risk.

**Interesting implications**

The results above give insight into what really goes on inside the CAPM. It should be remembered that the CAPM is an ingenious equilibrium model, and has internal dynamics that are not always appreciated, or even mentioned. The fact that asset beta is influenced jointly by the mean payoff and payoff covariance is critically important to a proper understanding of the CAPM, especially in applications such as capital budgeting.

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The following points follow from what has been found and should become part of our knowledge of corporate finance.

1. **Factors that don’t seem to be risks in any sense can materially affect the CAPM market risk premium.** Any consideration that increases the payoff expectation from a project while not adding proportionately to payoff covariance should result in a reduced CAPM discount rate. Clearly, therefore, a simple improvement in profitability can lead to a lower discount rate. That seems a remarkable statement, but it is not hard to show. Imagine a cash payoff of \( C + x \) where only \( x \) is random. As the constant \( C \) increases, the asset effectively becomes risk free, but its variance and covariance with the market are unchanged. Common sense and the CAPM both suggest that its discount rate will approach the risk-free rate as \( C \) gets larger. Critically, that increase in \( C \) reduces the CAPM discount rate — not by changing the payoff covariance, but by increasing the mean payoff. Ultimately, as \( C \to \infty \), the asset becomes risk-free, its expected return approaches the risk-free rate and its returns covariance approaches zero.

2. **Risks that might seem to be idiosyncratic or ‘unpriced’ are really not.** For example, suppose the firm is considering a change in CEO. In textbooks that would be regarded as a firm-specific or idiosyncratic uncertainty or risk, because it seems to affect only the single firm and hence can be ‘diversified away’. According to a naive understanding of the CAPM, the firm will not be penalised for such uncertainty, because it is a uniquely firm-specific variable. The obvious truth, however, is that a new CEO, or merely an increased probability of a new CEO, will influence both the expected cash payoff and the payoff covariance, in different proportions, and hence cannot be expected to leave their Fama ratio unchanged (e.g. suppose that it is known that any new CEO will almost certainly terminate a costly and unsuccessful venture that has been protected by the current CEO). This same argument holds for essentially every supposed ‘idiosyncratic risk’ (e.g. the risk that the new oil field is dry, the risk that the firm will suffer from a court decision or an industrial accident).
3. Suppose that the firm buys into an oil well. That might seem to be risky, but not be a systematic risk. First, it is a priced risk for the firm, because it does affect the Fama ratio. Second, and even more interestingly, if the increase in mean payoff outweighs any increase in firm payoff covariance then going into oil will actually drive the firm’s rational CAPM cost of capital downwards. That seems paradoxical, but follows immediately from Fama’s argument.

The very same point can be shown in the language of returns rather than payoffs. Specifically, going into oil can reduce the firm’s returns covariance with the market. Suppose that the firm announces its decision and its stock price changes from $P$ to $P_{oil}$, due to the market revising the firm’s mean payoff, from $E[V]$ to $E[V_{oil}]$, and similarly its payoff covariance from $\text{cov}(V, r_M)$ to $\text{cov}(V_{oil}, r_M)$. By mathematical definition, its forward-looking returns covariance $\text{cov}(r, r_M)$ changes then from $\text{cov}(V, r_M)/P$ to $\text{cov}(V_{oil}, r_M)/P_{oil}$. Hence, its returns covariance is reduced if

$$\frac{\text{cov}(V_{oil}, r_M)}{P_{oil}} < \frac{\text{cov}(V, r_M)}{P}$$

which requires simply that any increase in payoff covariance brought by the new investment is outweighed by a simultaneous increase in the CAPM price of the firm. This can happen when the market perceives an increase in payoff covariance along with a more than commensurate increase in payoff mean, with the net effect that the stock price goes up enough to satisfy the stated condition.

Note that it is easily shown by substituting for $P$ and $P_{oil}$, using the payoffs form of the CAPM above, that the necessary condition for the returns covariance to fall is

$$\frac{\text{cov}(V_{oil}, r_M)}{E[V_{oil}]} < \frac{\text{cov}(V, r_M)}{E[V]}$$

Thus, as we already know, the firm’s discount rate falls if its Fama ratio falls. The beauty of writing this condition in terms of the Fama ratio, rather than in terms of the firm’s returns, is that we go straight to the firm’s business fundamentals, namely the forward-looking mean and covariance of its cash payoff.

4. A change in activities can affect a firm’s cost of capital, and yet have only a negligible effect on the overall market average cost of capital. This indicates that single business ventures or even firms make up such a tiny part of the aggregate market that changes in their individual means and covariances have a negligible effect on the mean market payoff and variance of the market payoff. At the individual asset level, it seems reasonable to conclude that there is no possible change in business operations (e.g. new CEO, product mix, factory design) that will not affect the individual firm’s CAPM cost of capital. Put another way, changes in business operations have no effect on the firm’s forward-looking CAPM cost of capital or discount rate if and only if the Fama ratio is unchanged. In reality, that cannot ever occur since any change of substance will affect either or both the perceived payoff mean or covariance.

5. A common mistake is to presume that firm-specific considerations that randomise out at the market aggregate level do not add to the individual firm’s cost of capital (i.e. they are not ‘priced’ at the firm level). For example, suppose that the firm invests heavily in a new product. If successful, the firm will take a bigger market share and some of its competitors will lose sales. In a textbook sense, this is a ‘diversifiable’ or firm-specific risk, and does not materially affect the market aggregate payoff mean or variance. That does not suggest, however, that the new product is irrelevant to the firm’s discount rate. Rather, it may greatly alter the firm’s forward-looking mean payoff and payoff covariance, and therefore have a significant effect on the firm’s cost of capital.
Conclusion
This paper shows how much we can learn about the CAPM by examining its pricing or ‘certainty equivalent’ form, instead of its more conventional and less intuitive ‘returns form’. The usual driver of the CAPM discount rate is taken to be ‘beta’ (i.e. the returns beta), and that still holds. But when we switch our thinking into the language of cash flows or payoffs, the driver of beta and of the CAPM cost of capital is revealed to be Fama’s ratio, namely the ratio of mean payoff to payoff covariance. This finding is critically important in applications for which the payoffs CAPM is the natural way to think. These include capital budgeting and all of the firm’s investment decisions under uncertainty.

The key proposition in this paper may seem to contradict the CAPM, but it really only clarifies the CAPM. A natural question is to ask which finance theorist told us that an increase in the expected payoff from a project can in and of itself warrant a decrease in its CAPM discount rate? The answer is that Fama (1977) did. Unfortunately, his analysis and explanation was not given the attention it deserved and, until recently, it has been forgotten.

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Why that happened is a mystery. Part of the reason is that Fama’s argument requires us to view the CAPM as an equilibrium mechanism, and it is relatively difficult to explain when compared to the simpler notion that a firm has a ‘beta’ which drives its risk premium. Undoubtedly, that complexity partly explains why our interpretation of the CAPM has been a little oversimplified. The other more obvious reason is that in all areas of finance where stock returns are effectively exogenous, being generated by a stock market rather than by a business venture (e.g. hotel) that we design and build ourselves, it seems obvious to think about the usual returns form of the CAPM, in which case the returns beta’s determinants are a secondary concern. In effect, stock market returns are treated as outputs of an ‘uncontrollable’ exogenous stochastic process, much like the weather.

Fama’s argument is most relevant to capital budgeting, where the payoffs form of the CAPM applies naturally, and is often held to be the correct way to work out CAPM asset values. In effect, Fama revealed that this form of the CAPM implies interesting and little-known principles about how to understand the discount rate applicable to a new venture. Methods that discount the expected payoff from a business at one constant rate (e.g. a firm hurdle rate or WACC), even when the amount of that expected payoff changes, are generally incorrect in terms of CAPM principle. Instead, a change in the numerator (expected cash payoff) demands a change in the denominator (discount rate). A related but less surprising argument is set out by Kruger et al. (2015).
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Note
1. This idea was suggested to me by Tony van Zijl, Professor of Accounting and Financial Management, Victoria University of Wellington.

References
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