Value at risk: par or zero?

How to get the best out of your risk measurement tools

Properly constructed, interest rate VAR models are a useful source of information. PAUL SMETANIN explains that when comparing par-based and zero-based models, it becomes clear that the par-based model is superior.

Financial institutions have increasingly been interested in the measurement of the market risk that they carry, a process reinforced by the Basle initiatives towards establishing a market risk capital framework. As part of these new developments the concept of “value at risk” has been given wide acclaim as being the “best practice” calculation of market risk. In Australia, this is reinforced by the Reserve Bank’s draft Prudential Statement C3 Capital Adequacy of Banks: Market Risk which specifically supports the use of value-at-risk techniques.

Value at risk (VAR) is a measure of a bank’s exposure to loss, normally on a daily basis. The calculation of VAR can take many forms, the most popular being statistical-based models. Statistical VAR uses the same techniques as those used in option pricing and asset allocation. Its primary foundations have been known since 1952 when Markowitz developed the idea of minimum variance portfolios.

This article considers the application of statistical VAR in interest rate risk management. Specifically, it considers whether an interest rate VAR model should use zero coupon rates or par rates in the calculation of market risk. Why construct a VAR model if it is not useful for the day-to-day risk management process? It is argued that par rates should be used on the basis they are not only consistent with its theoretical counterpart, zero rates, but they provide superior interest rate risk management information.

RBA MODEL REQUIREMENTS
The RBA is extending its existing capital adequacy framework to those market risks associated with bank trading activities. The initiative is partly designed to prompt banks to become increasingly market-risk literate. This is evident from the draft statement’s requirement that the internal model should be:

- closely integrated with the bank’s day-to-day risk-taking activity;
- reconcilable to the bank’s internal risk-limit structure; and
- well understood by both traders and management.

In this way, the model must not only be a good measure of market risk, but be capable of providing information that is consistent with the risk-taking activities of the bank. As a consequence, a VAR model should conform with the following criteria if it is going to be embraced by banks as a risk-management tool:

- good measure of market risk by predicting a likely loss accurately within a given confidence interval;
- consistent with the daily profit-and-
loss results of the bank;
- consistent with the way risk is bought and sold in the market; and
- sufficiently simple to be understood by its users.

These criteria are concerned with the way the model should be developed. For an interest rate VAR model, they are primarily concerned with whether par or zero rates should be used as model inputs. The choice determines the model’s shape and ultimate usefulness.

A brief summary of the essential components of a VAR model is provided in Figure 2.

**ZERO RATES AND PAR RATES**

In debt markets, interest rates exist mainly as par rates. Par rates are the yield of a security that, when used to discount each cashflow in the security, provides the security’s present value. In this way, when par rates are quoted they include a coupon assumption. When securities are trading at par their present value is equal to their face value as the time value of money is exactly matched by the coupon cashflows.

A zero rate measures the true time value between a cashflow’s future value and its present value. Unlike par rates, there are no intervening cashflows assumed to exist. Zero rates rarely exist in isolation from par rates. They generally exist only for short-dated securities. As markets do not trade zero rates for longer-dated securities, they must be derived or inferred from par rates. The generation of zero rates from par rates can be achieved by the process of bootstrapping.

The question of whether zero or par rates should be used in a value-at-risk model begins with the independence of the data itself. There are essentially two types of data independence, linear and statistical. Linear independence requires that the movement of one data point must not be a function of the movement of another. Statistical independence requires that the movement of one data point must be uncorrelated with the movement of the other. VAR models require that data be linear independent only, as statistical dependence can be remedied by adjusting for any correlation that exists between the two data points.

Some have claimed that par rates cannot be used in VAR models as they do not possess the requisite independence. It is argued in response that zero rates, expressed mathematically, are not dependent on any other points along the yield curve – remembering that once calculated, a zero rate represents the pure time value for a particular future cashflow. Therefore they qualify for use in a statistical VAR model. As par rates represent a combination of cashflows and time values, it is argued that they do not qualify (theoretically) for use in a statistical VAR model because the movement of a par rate involves the movement of many interest rates along the yield curve.

Zero rates are derived from bootstrapping a par curve. The bootstrapping process is a form of interpolation. It therefore follows that zero rates derived from the bootstrapping process are not linearly independent. Par rates represent a single price for a marketable parcel of cashflows. Practitioners argue that par rates move independently of each other in that the pricing of a par rate does not rely upon other par rates. The fact that par rates represent a combination of cashflows does not detract from the market activity of pricing such securities independently.

Another argument raised in favour of zero rates is their use by many banks as a valuation tool. Many banks, if not all, generate zero rates to value the cashflows of their portfolio. Zero rates are useful in this process because daily discount factors can be generated to discount the many cashflows of the different portfolios of the bank. As one of the criteria of a good VAR model is that it must be consistent with the daily profit-and-loss activity of the bank, it is argued that zero rates must be used in its calculation of market risk. This is not necessarily the case.

As zero rates can be used for valuation, so too can par rates, remembering that zero rates come from par rates. For example, assume that a bank has only one security, a two-year bond with quarterly coupons. The bank’s valuation of the bond begins by using the bond’s yield to maturity to determine zero rates for the valuation of each cashflow in the bond. Alternatively, the bond could have been valued by taking its yield to maturity and discounting each cashflow by this average yield. The two approaches are exactly equal as their starting points are the same. Accordingly, the use of either zero rates or par rates will be consistent with the bank’s daily valuation process.

**ZERO AND PAR RATE BASED MODELS**

The ultimate objective of a VAR model is to calculate a likely loss in a day for a given confidence interval. The question remains whether a zero-based model or a par-based model generates consistent results. If they do generate consistent results, many of the technical arguments become irrelevant.

To examine the differences between the two approaches, a zero-based model and a par-based model were constructed, each with 17 grid points. Random par

<table>
<thead>
<tr>
<th>Using alternate short and long positions of $100m along the 17 grid points</th>
<th>Largest par volatility</th>
<th>Largest zero volatility</th>
<th>Difference</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest par volatility</td>
<td>$2,098,433</td>
<td>$2,099,809</td>
<td>$1,376</td>
<td>0.07%</td>
</tr>
<tr>
<td>Smallest zero volatility</td>
<td>$1,238,138</td>
<td>$1,237,295</td>
<td>$843</td>
<td>0.07%</td>
</tr>
</tbody>
</table>
rates were generated from which zero rates were derived. Using 50 different portfolio simulations with random par curve changes, the results of the two models were compared. The results are summarised in Table 1.

The average difference was $281 (0.02%) and the standard deviation of differences was $2,034 (0.13%).

The tests show that the two models are consistent in the results they generate. Any differences are attributed to the fact that averages (volatility and covariance) are being applied to specific position sensitivities (ie, present value basis point) on a particular day.

These results should not be surprising as both models have the same starting points, being par rates and original cashflows. The equality between the two models can be represented as in Figure 1.

What is done to one side is done to the other, so that mathematical symmetry is maintained, ensuring the results from the two models are consistent.

**VAR MODELS AND RISK MANAGEMENT**

The risk management systems of banks should be consistent with the way risk can be bought and sold in the market. This is a basic necessity as the management of the bank requires information in the same form, although summarised, as it is generated by the risk-taking activities. In this way, as a risk-management tool, yields are kept in the form in which they exist in the market, that is, as par rates.

As a valuation tool, par yields are generally not used and are instead broken into zero coupon yields to allow the valuation of discrete cashflows of a large portfolio. This is a flexibility consideration, not a matter of necessity.

The criteria that define a good VAR model require that it be both a valuation tool (measuring likely loss) and a risk-management tool (sourcing risk). This is where par-based models have the advantage.

A zero-based model takes par securities and breaks them down into cashflows. These cashflows are then manipulated in a way that generates VAR. In order to source the contribution of a par product to the overall VAR result is near-impossible, as its cashflows have been disassociated in the modelling process. In this way, a zero model is inconsistent with the way risk is bought and sold in the market.

A par-based model maintains the association between cashflows in a way that is consistent with what is traded in the market. As cashflows are kept together, the sourcing of a product’s VAR contribution is easily achieved as it can be traced through the covariance matrix.

The implications of these differences are:

- a par-based VAR model would be consistent with the bank’s internal treasury limit structure as it would retain market conventions (eg, limit the number of futures, swaps etc). A zero-based model would require a cash-based limit structure which would be difficult to administer;
- par-based models are easier to manage as manipulation of the VAR result can be achieved by using par products. In zero-based models the relationship is not so obvious; and
- par-based models are easier to understand as products are not broken down, whereas a zero-based model requires all products, including derivatives, to be broken down. Information presented using zero rates and cashflows is unfamiliar to the risk taker.

**CONCLUSION**

Interest rate VAR models are a useful source of information provided that they are constructed correctly. When making comparisons between a par-based model and a zero-based model, it becomes increasingly obvious that given both are consistent, the par-based model offers much more.

Par-based models are a good measure of market risk, consistent with the daily profit and loss of the bank, represent an observable market price for cashflows that can be traded, are easily incorporated in the risk-management process, easily understood and familiar to the eyes of market participants. These are the hallmarks of a good model.
A value-at-risk model is a process of determining the exposure of a bank to a loss over a given time-frame for a given confidence interval. If a statistical approach is adopted for the basis of the model, the main components can be illustrated as in Figure 2.

Position data represents the actual positions that the bank holds. It is the position data that ultimately determines the exposure of the bank to movements in market rates.

Price data is collected from commonly traded points along the yield curve. Price data forms the basis of the volatility of each grid point and the covariances between grid points.

Grid point allocation is the result of summarising the complexity of the position data across discrete points along the yield curve using certain conditions (eg, maintaining present value, sensitivities, etc). Grid points allow the calculation to be matched against yield curve points of interest, usually traded points.

Position sensitivities is the sensitivity of each yield curve grid point for a one basis point change in yield. This allows positions to be expressed in terms of sensitivity which, when combined with volatility, generates a value change likely for each grid point.

Volatility is a measure of the variability of price data at each grid point. The measure of volatility usually takes the following form:

\[ \sigma = \left[ \frac{\sum (r_i - r_e)^2}{N} \right]^{1/2} \]

Where \( r_i \) is the change in data for a given day expressed in return terms, \( r_e \) is the expected change in data for a day. \( N \) is the number of data points used in the calculation.

Covariance measures the degree of relationship between two grid points. On its own, covariance has no descriptive qualities apart from being useful in the value-at-risk calculation. Correlation is derived from covariance which measures the degree of relationship between two grid points by using a scale of -1 to +1. The measure of covariance usually takes the form of:

\[ \text{Cov}_{i,j} = \frac{\sum (r_{i} - r_{e}) \times (r_{j} - r_{e})}{N} \]

Where \( r_i \) is the actual change in data point 1, \( r_j \) is the expected change in data point 1, \( r_i \) is the actual change in data point 2 and \( r_j \) is the expected change in data point 2. The blending of more than 2 grid points is a simple extension of the formula.

The covariance matrix blends the position sensitivity, volatility and covariance information in a way that determines the collective distribution of portfolio value. This blending process calculates portfolio variance in accordance with the following formula:

\[ \sigma_p^2 = S_1^2 \times q_1^2 + S_2^2 \times q_2^2 + 2 \times S_1 \times S_2 \times \text{Cov}(r_i, r_j) \]

This formula blends two grid points. \( S_1 \) is the position sensitivity of grid point 1, \( S_2 \) is the position sensitivity of grid point 2.

The square root of the portfolio variance is the portfolio's standard deviation. It is the portfolio standard deviation that is the basis of the VAR figure. If the model back-tests well and is consistent with the RBA's qualitative and quantitative standards, then VAR will be:

\[ \text{SVAR} = \text{portfolio standard deviation} \times 2.33 \times 3 \]

The gross-up of 2.33 is the number of standard deviations required in order to cover 99% of the portfolio value changes under the assumption of normality. The gross-up of 3 is the minimum RBA gross-up of the statistical 99% result to account for the deficiencies of the statistical calculation. This \( \text{SVAR} \) is for a one-day liquidation period only. The RBA will require a 10-day liquidation period. The 10-day \( \text{SVAR} \) can be estimated by multiplying the one-day \( \text{SVAR} \) by the square root of 10.